Morphological Image Processing

Preview

- Morphology “ – a branch in biology that deals with the form and structure of animals and plants.

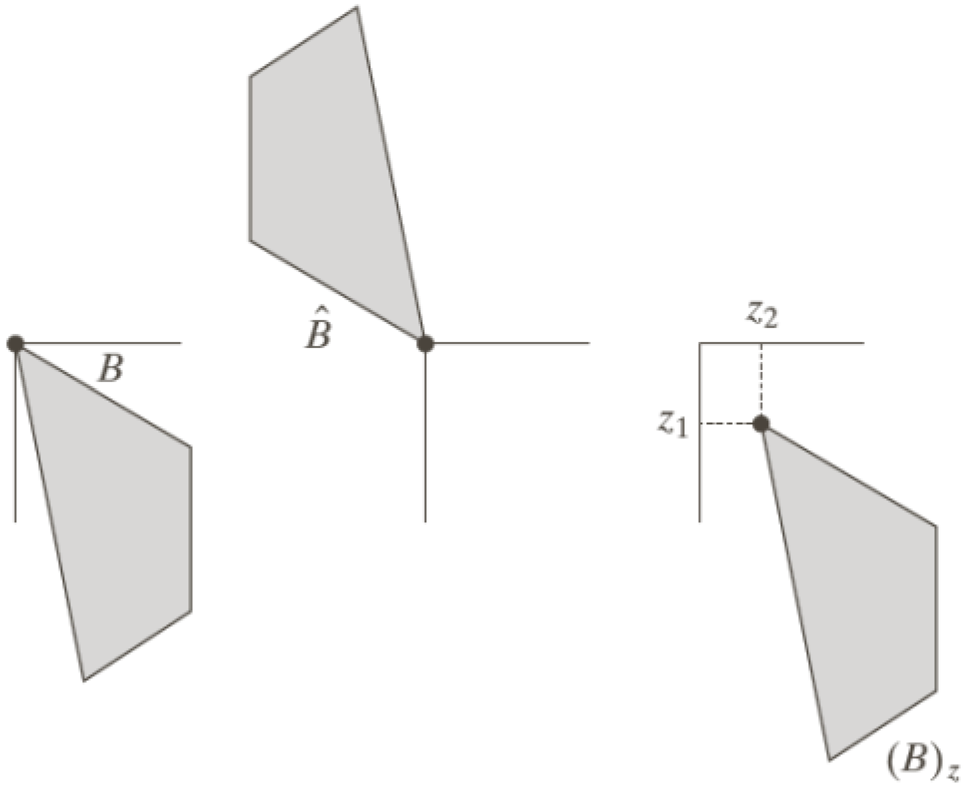
- Mathematical morphology is used to extract some properties of the image, useful for its presentation and descriptions. For example, contours, skeletons and convex hulls.

Preliminaries

Reflection and translation

$$\hat{B} = \{w \mid w = -b,\ for\ b \in B\}$$

$$(B)_z = \{c \mid c = b + z,\ for\ b \in B\}$$



Structuring Elements

- Morphological operations are defined based on structuring element.
- A structuring element is a small sets or sub images used to probe an image under study.

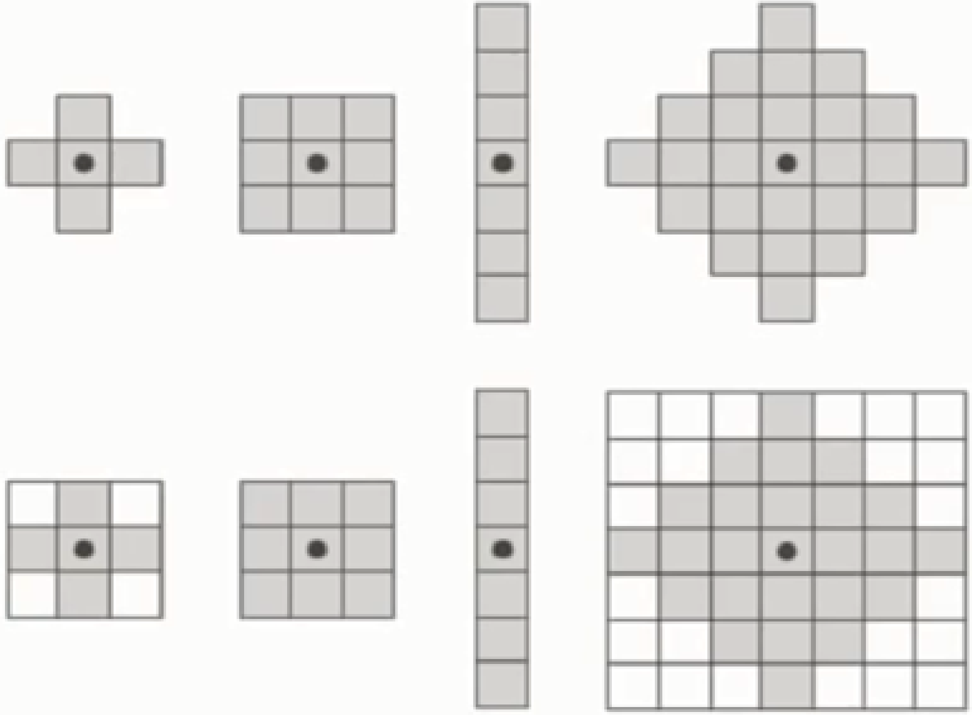


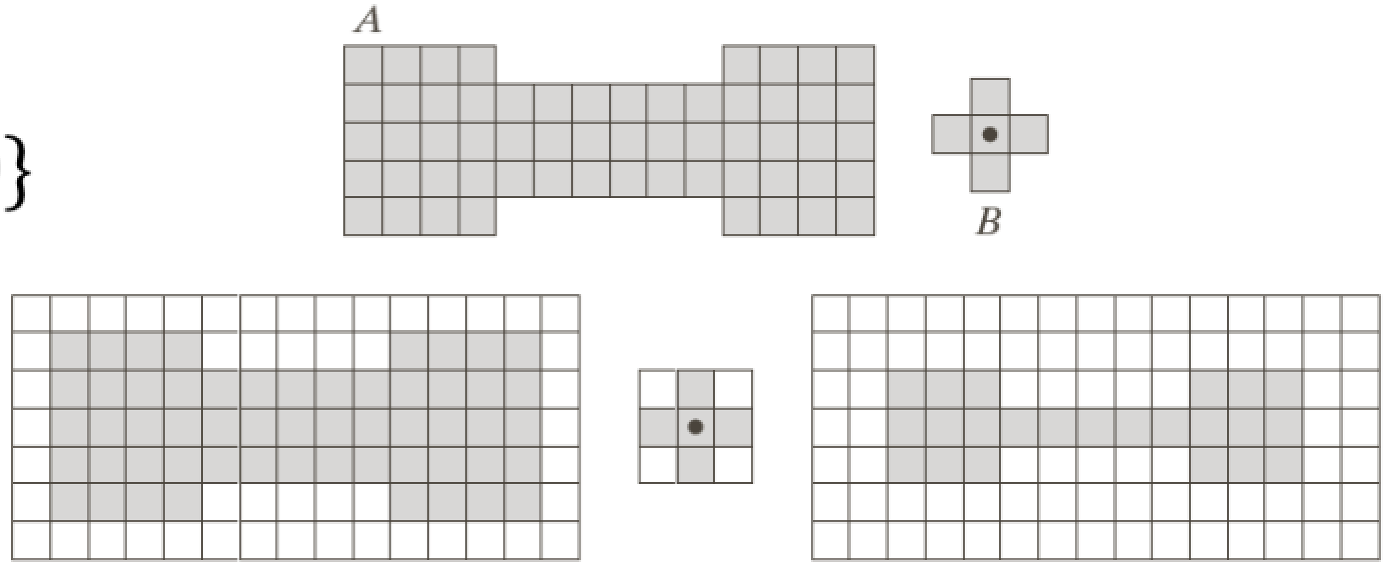
FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

Erosion

- Erosion is shrinking or thinning operation
- The erosion of A by B is denoted by $A \ominus B$

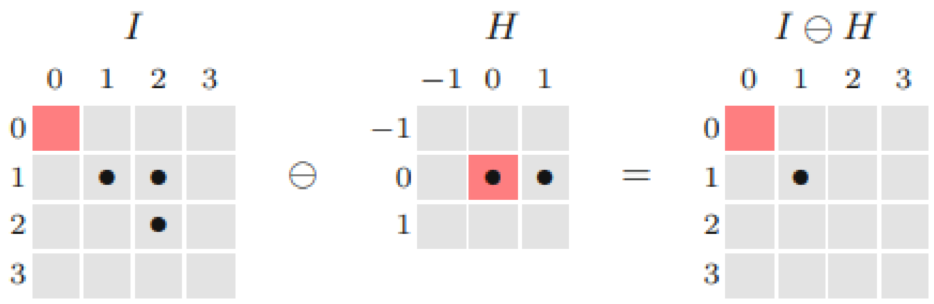
$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

$$A \ominus B = \{z \mid (B)_z \cap A^c = \emptyset\}$$



Erosion

$$I \ominus H \equiv \{\boldsymbol{p} \in \mathbb{Z}^2 \mid (\boldsymbol{p}+\boldsymbol{q}) \in I,\ \text{for all } \boldsymbol{q} \in H\}.$$



$$I \equiv \{(1,1),(2,1),(2,2)\},\ \ H \equiv \{(\mathbf{0},\mathbf{0}),(\mathbf{1},\mathbf{0})\}$$

$$I \ominus H \equiv \{\,(1,1)\,\}\ \text{ because}$$

$$(1,1)+(\mathbf{0},\mathbf{0})=(1,1)\in I \quad \textbf{and} \quad (1,1)+(\mathbf{1},\mathbf{0})=(2,1)\in I$$

Erosion

1	1	1	1	1	1
1	1	0	0	1	1
1	0	0	0	0	1
1	1	0	0	1	1
1	1	1	1	1	1

A

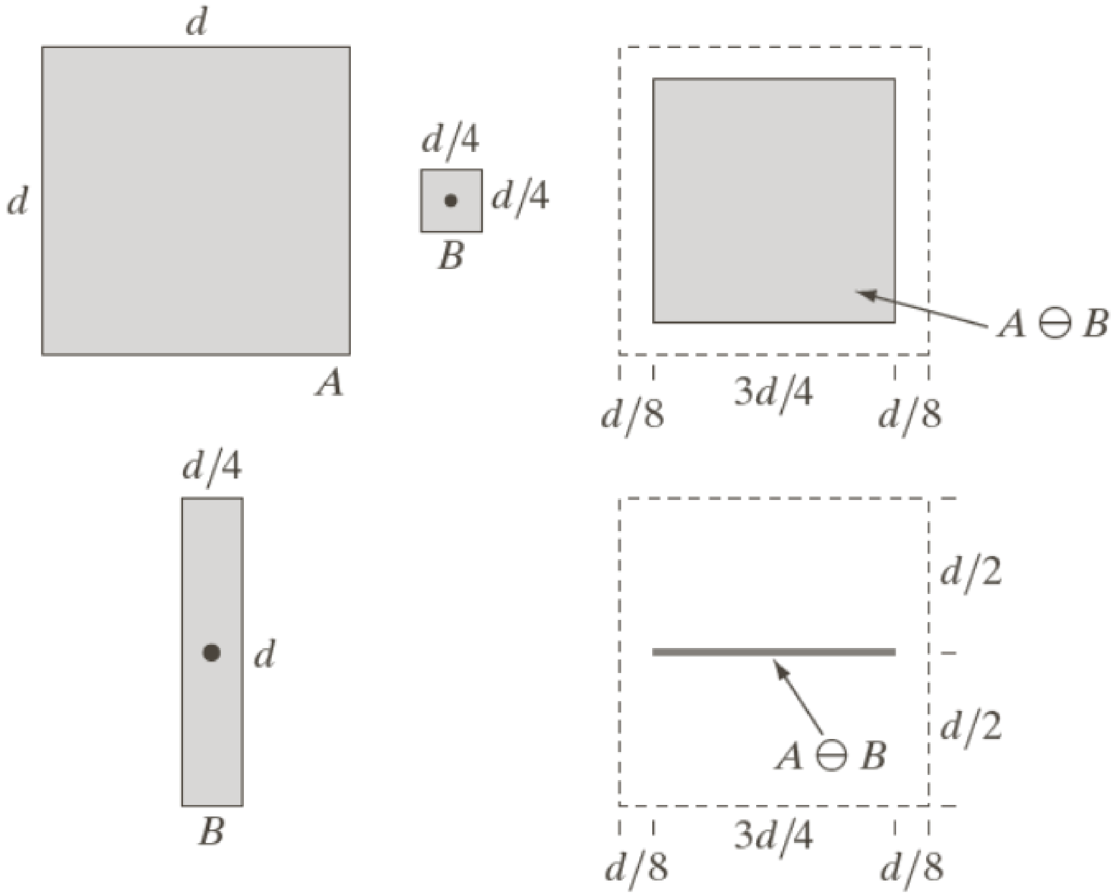
1
1
1

B

0	0	0	0	0	0
1	0	0	0	0	1
1	0	0	0	0	1
1	0	0	0	0	1
0	0	0	0	0	0

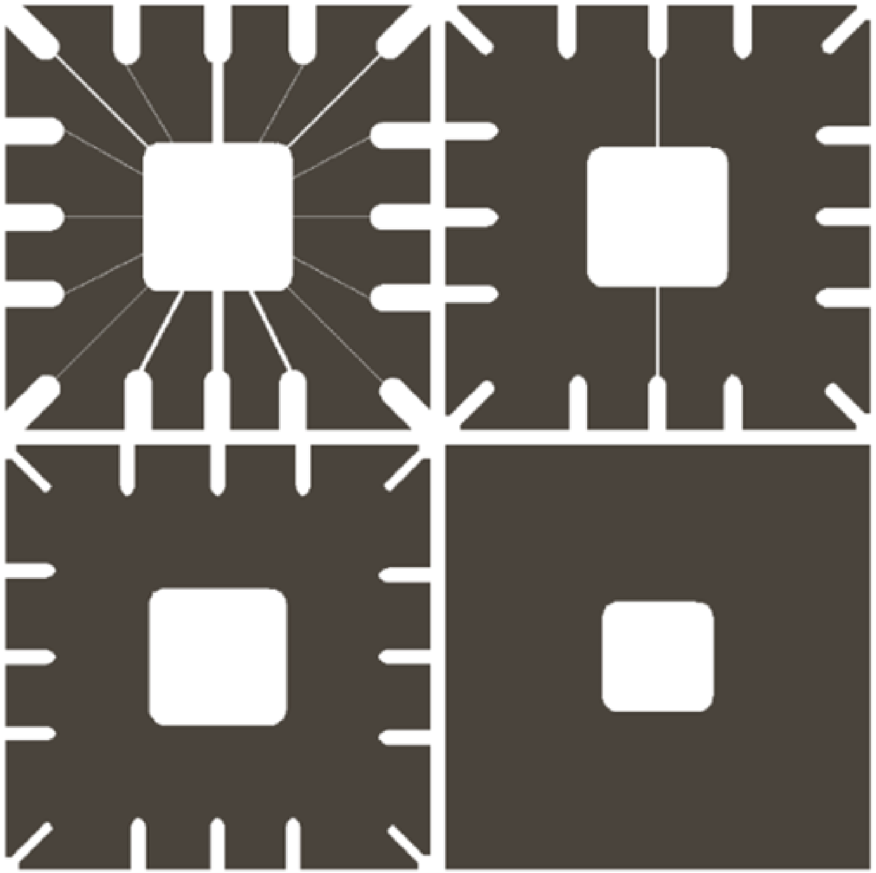
$A \ominus B$

Erosion Example



Erosion Example

Image eroded using square structuring elements of sizes 11 × 11 , 15 × 15, and 45 × 45 elements, respectively, **all valued 1.**



Dilation

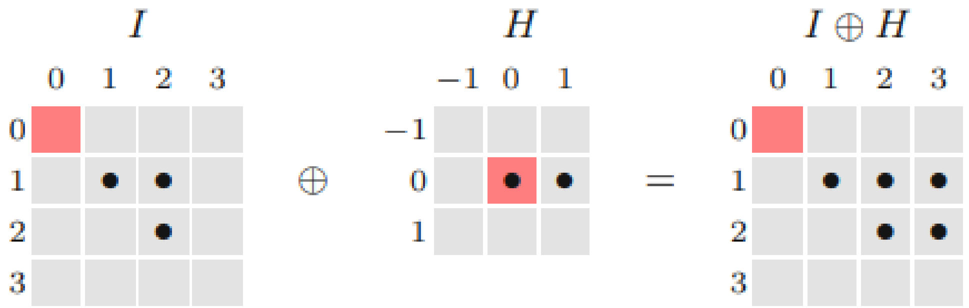
- Dilation grows or thickens object
- The dilation of A and B is set of all displacement z, such that $\hat{B}$ and A overlap by at least one element.

$$A \oplus B = \{z | (\hat{B})_z \cap A \neq \emptyset\}$$

$$A \oplus B = \{z | [(\hat{B})_z \cap A] \subseteq A\}$$

Dilation

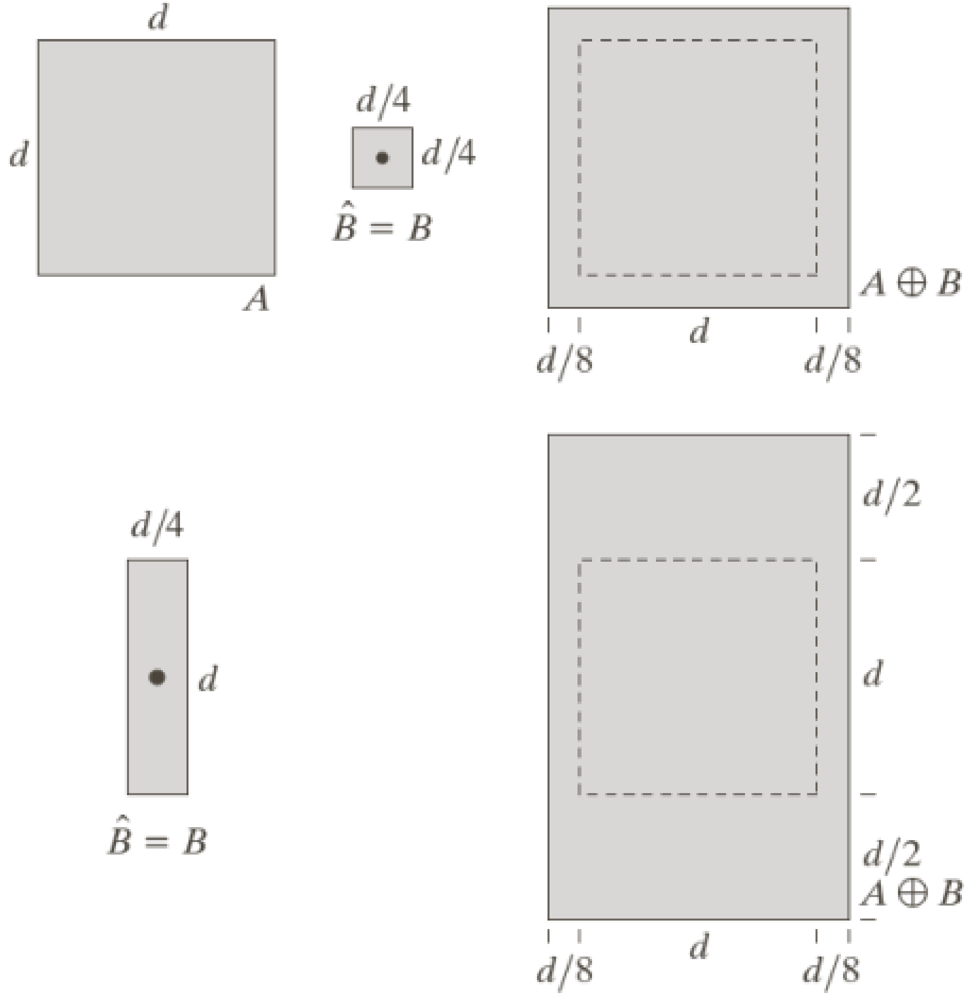
$$I \oplus H \equiv \left\{ (\boldsymbol{p} + \boldsymbol{q}) \mid \text{for all } \boldsymbol{p} \in I, \boldsymbol{q} \in H \right\}.$$



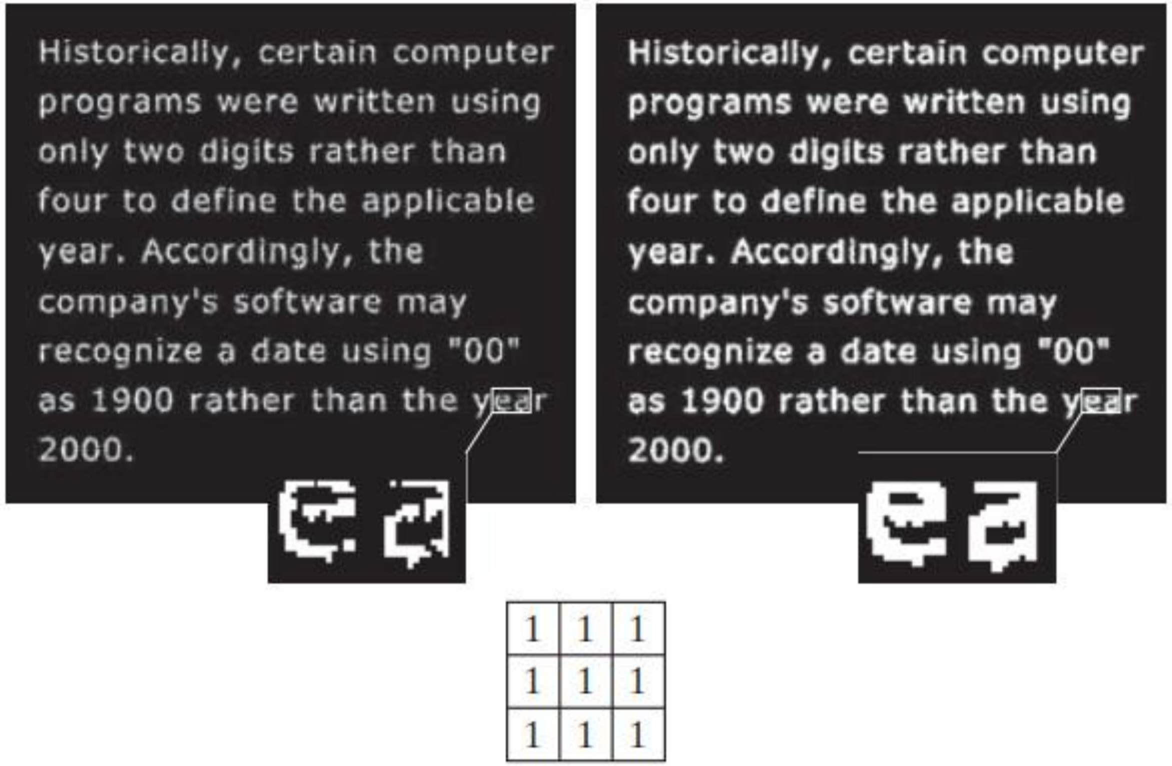
$$I \equiv \{(1,1),(2,1),(2,2)\}, \;\; H \equiv \{(\mathbf{0},\mathbf{0}),(\mathbf{1},\mathbf{0})\}$$

$$\begin{aligned} I \oplus H \;\equiv\; \{ \; & (1,1)+(\mathbf{0},\mathbf{0}),\; (1,1)+(\mathbf{1},\mathbf{0}), \\ & (2,1)+(\mathbf{0},\mathbf{0}),\; (2,1)+(\mathbf{1},\mathbf{0}), \\ & (2,2)+(\mathbf{0},\mathbf{0}),\; (2,2)+(\mathbf{1},\mathbf{0}) \; \} \end{aligned}$$

Dilation Example



Dilation Example



a c
b

FIGURE 9.7
(a) Low-resolution text showing broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Example of Dilation

0	0	0	0	0	0
0	0	1	1	0	0
0	0	0	0	1	0
0	0	1	1	0	0
0	1	0	0	1	0
0	0	0	0	0	0

Set A

Set B

1	1

Example of Dilation

0	0	0	0	0	0
0	0	1	1	0	0
0	0	0	0	1	0
0	0	1	1	0	0
0	1	0	0	1	0
0	0	0	0	0	0

Set A

Set B

1	1

1	1

Reflection of B

0	0	0	0	0	0
0	0	1	1	1	0
0	0	0	0	1	1
0	0	1	1	1	0
0	1	1	0	1	1
0	0	0	0	0	0

$A \oplus B$

Duality

- Erosion and dilation are duals of each other with respect to set complementation and reflection.

$$(A \ominus B)^c = \{z \mid (B)_z \subseteq A\}^c$$

$$(A \ominus B)^c = \{z \mid (B)_z \cap A^c = \varnothing\}^c$$

$$(A \ominus B)^c = \{z \mid (B)_z \cap A^c \neq \varnothing\} = A^c \oplus \hat{B}$$

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

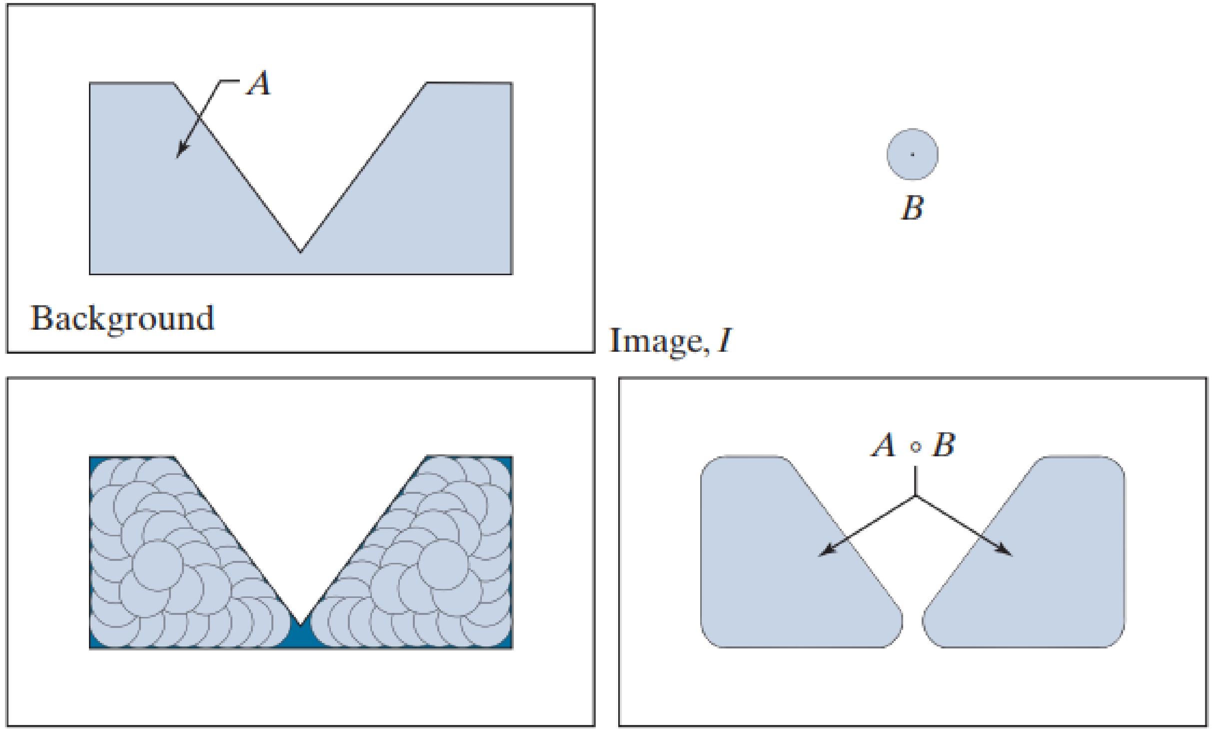
$$(A \oplus B)^c = A^c \ominus \hat{B}$$

Opening

- An erosion followed by dilation

$$A \circ B = (A \ominus B) \oplus B$$

the opening **of *A*** by *B* is obtained by taking the union of all translates of B that fit into ***A* .**

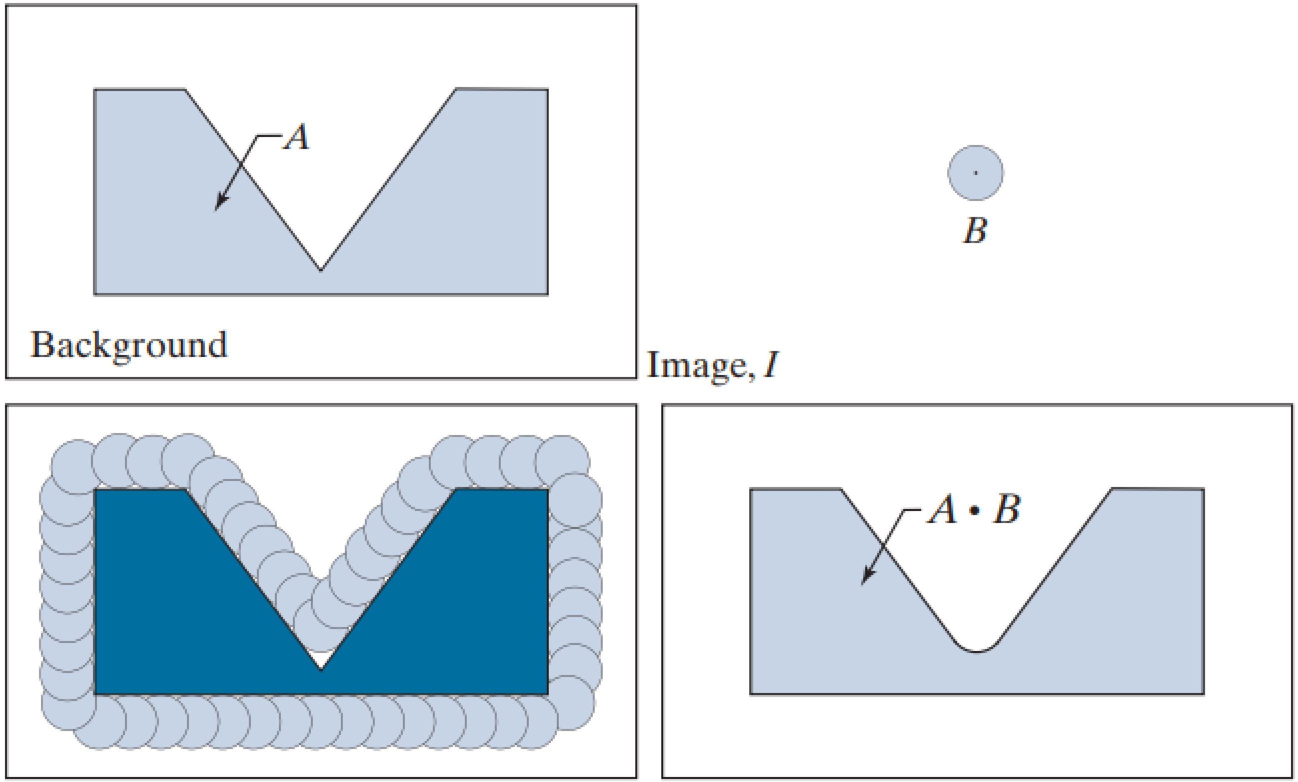


Closing

Closing is then the complement of the union of all translations of B that do not overlap A

- A dilation followed by erosion

$$A \bullet B = (A \oplus B) \ominus B$$



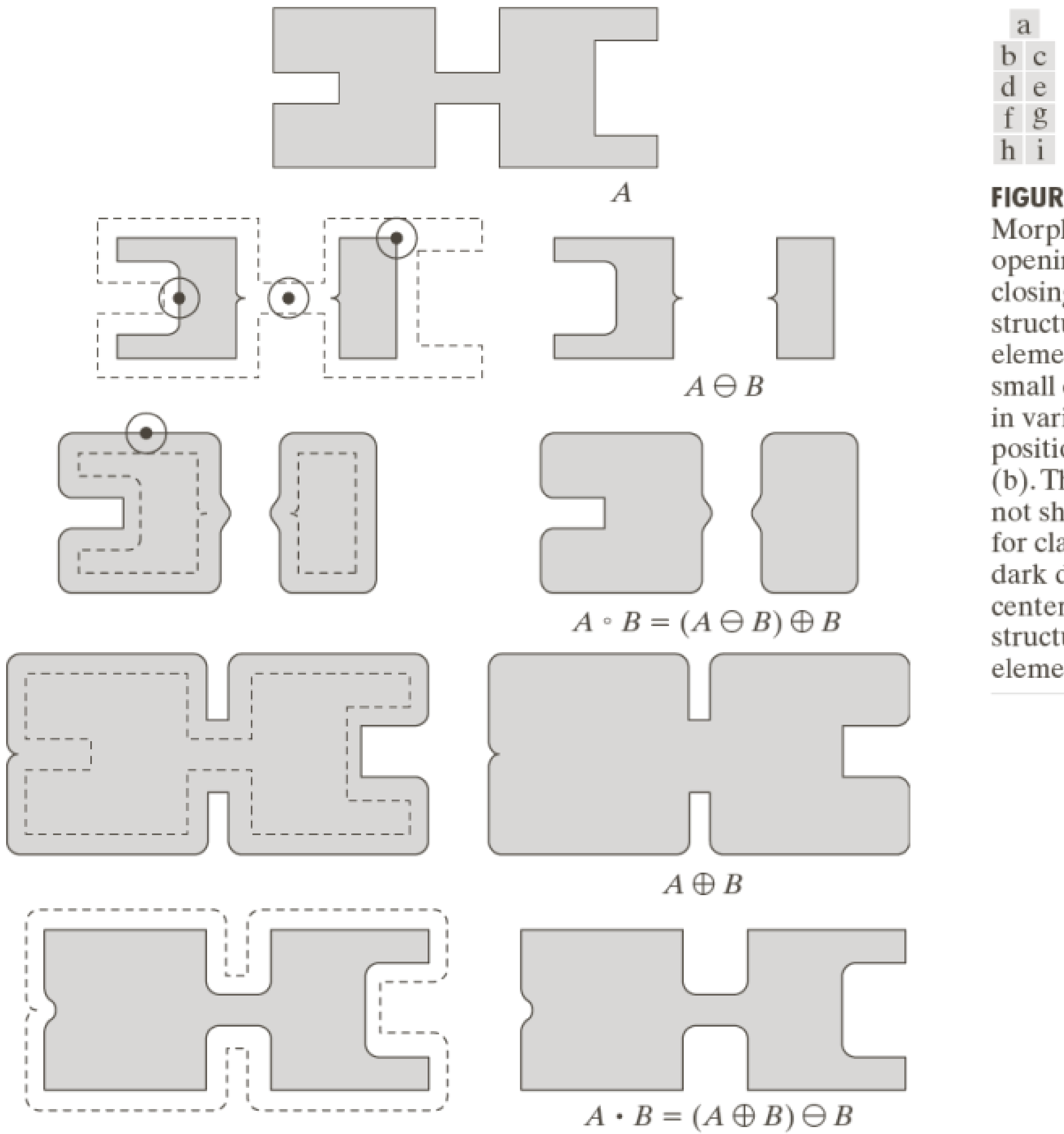


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of *A*.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

Morphological opening has the following properties:

(a) $A \circ B$ is a subset of A.

(b) If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.

(c) $(A \circ B) \circ B = A \circ B$.

Similarly, closing satisfies the following properties:

(a) A is a subset of $A \bullet B$.

(b) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$.

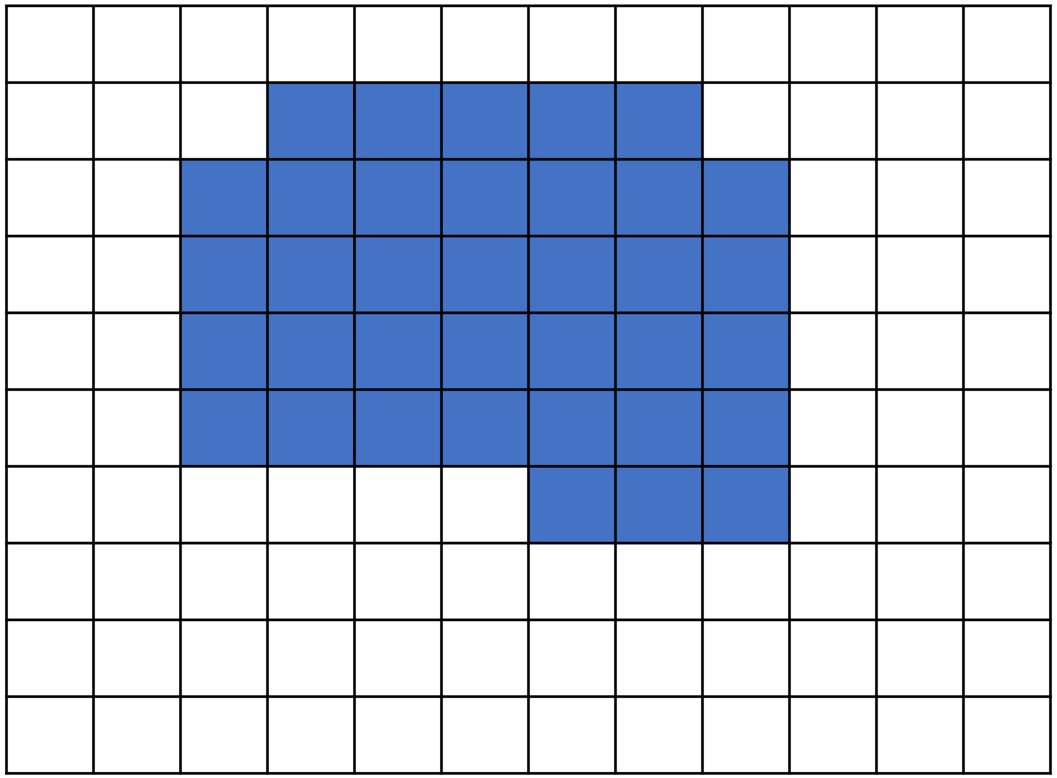
(c) $(A \bullet B) \bullet B = A \bullet B$.

Boundary Extraction

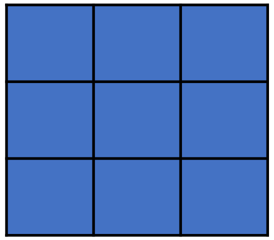
- Detect the boundary of an object region
- The boundary of a set A is denoted by $\beta(A)$
- If B is a suitable structuring element then

$$\beta(A) = A - (A \Theta B)$$

Boundary Extraction

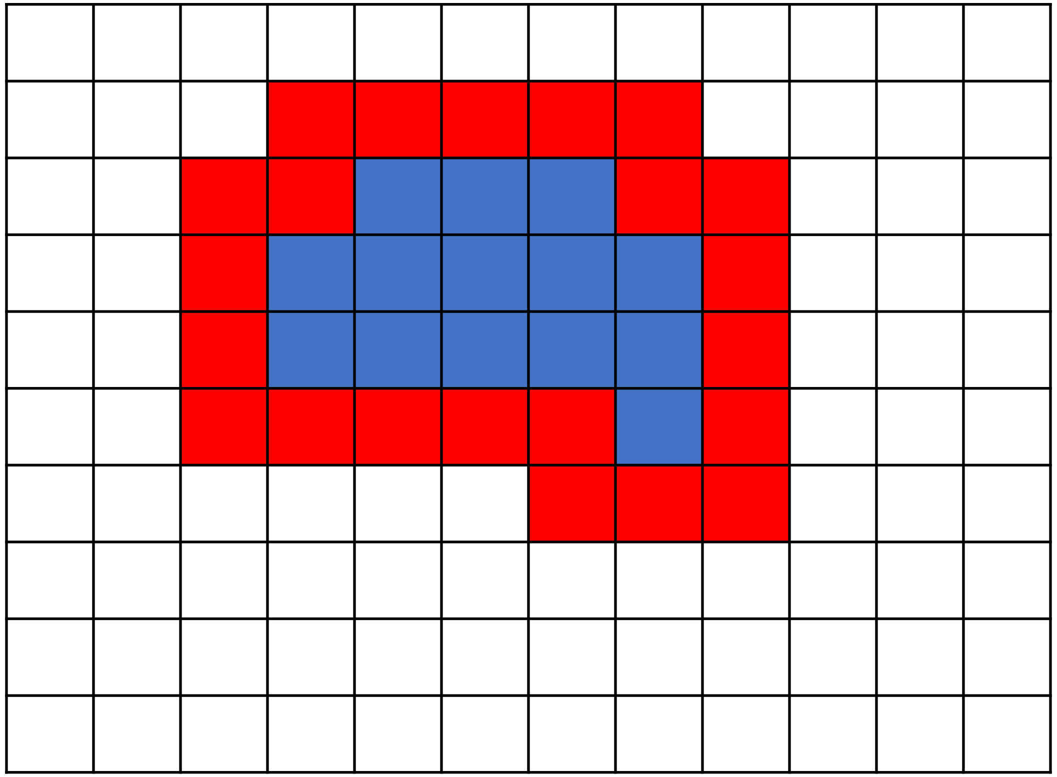


A

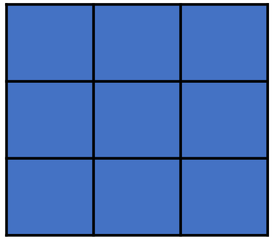


B

Boundary Extraction

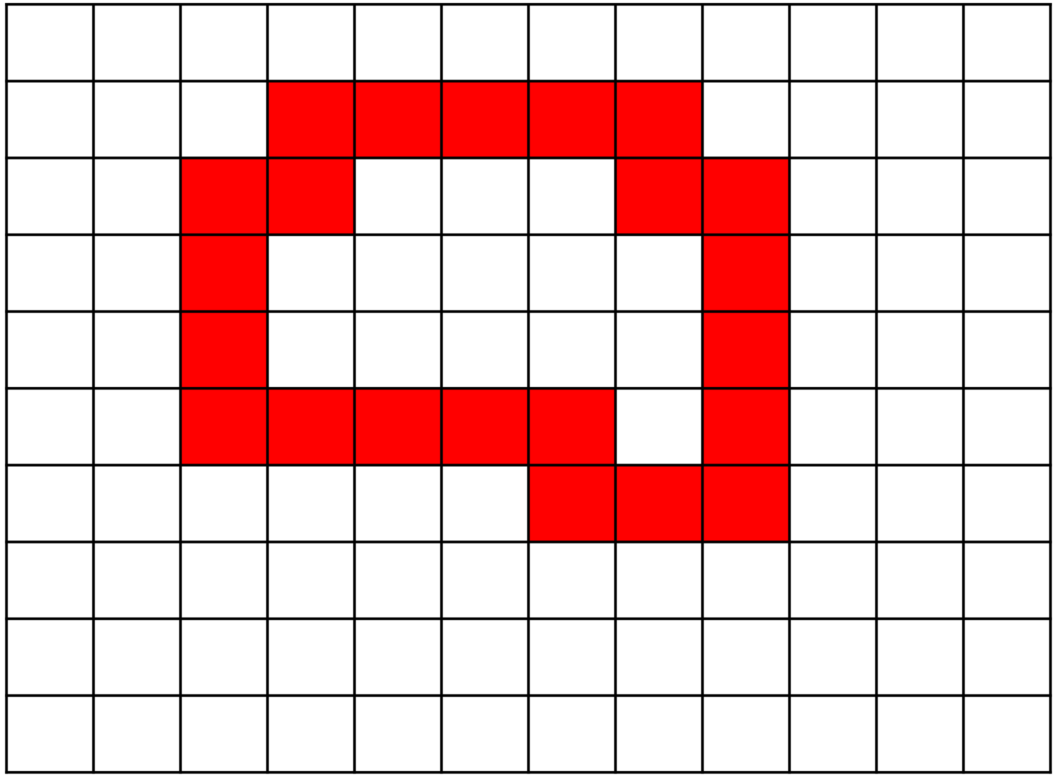


A ϴ B

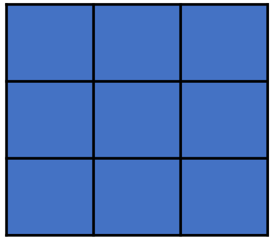


B

Boundary Extraction



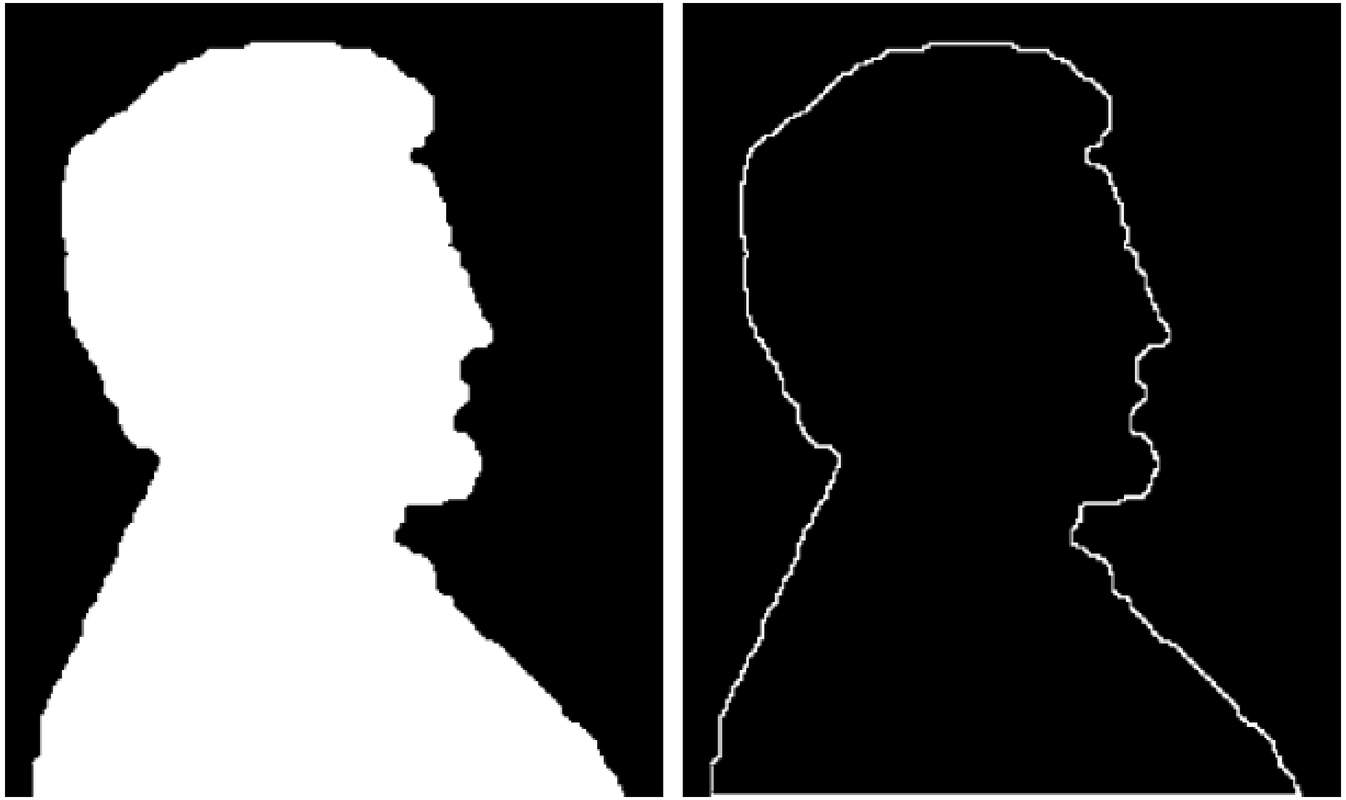
A – (A ϴ B)



B

Boundary of an Object denoted by β(A) = A - A ϴ B

Boundary Extraction



a b

FIGURE 9.14 (a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

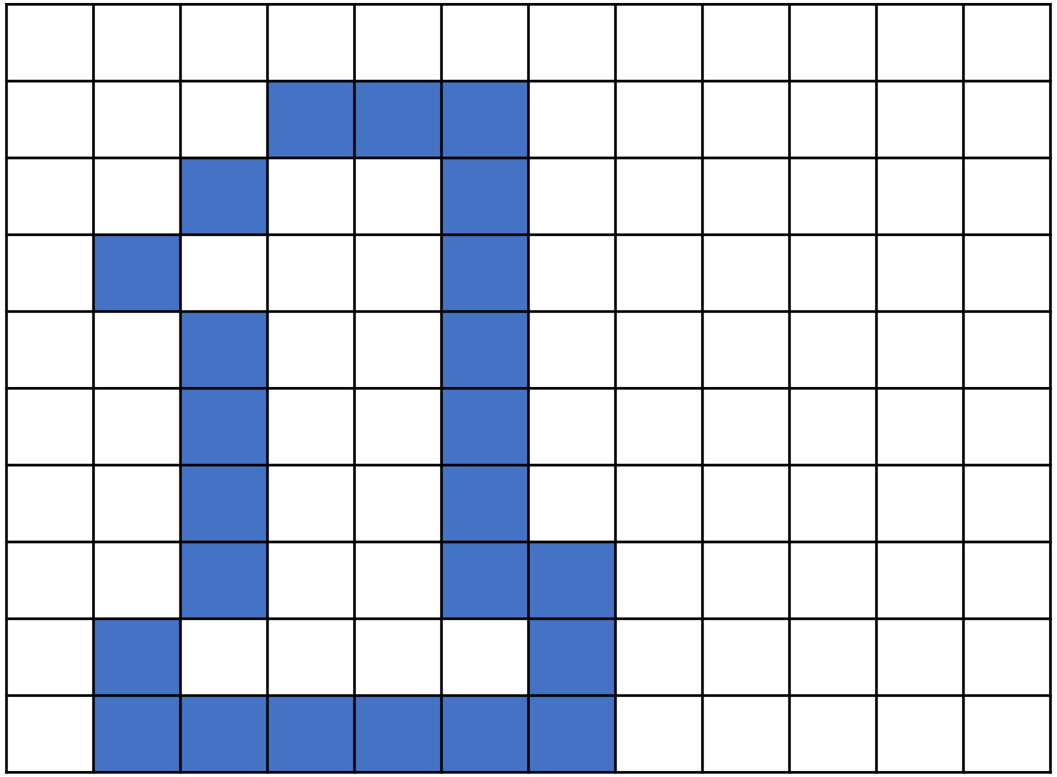
1	1	1
1	1	1
1	1	1

Hole Filling

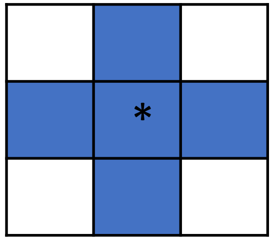
- Fill up the hollow region within a boundary

$X_k = (X_{k-1} \oplus B) \cap A^c$

Hole Filling

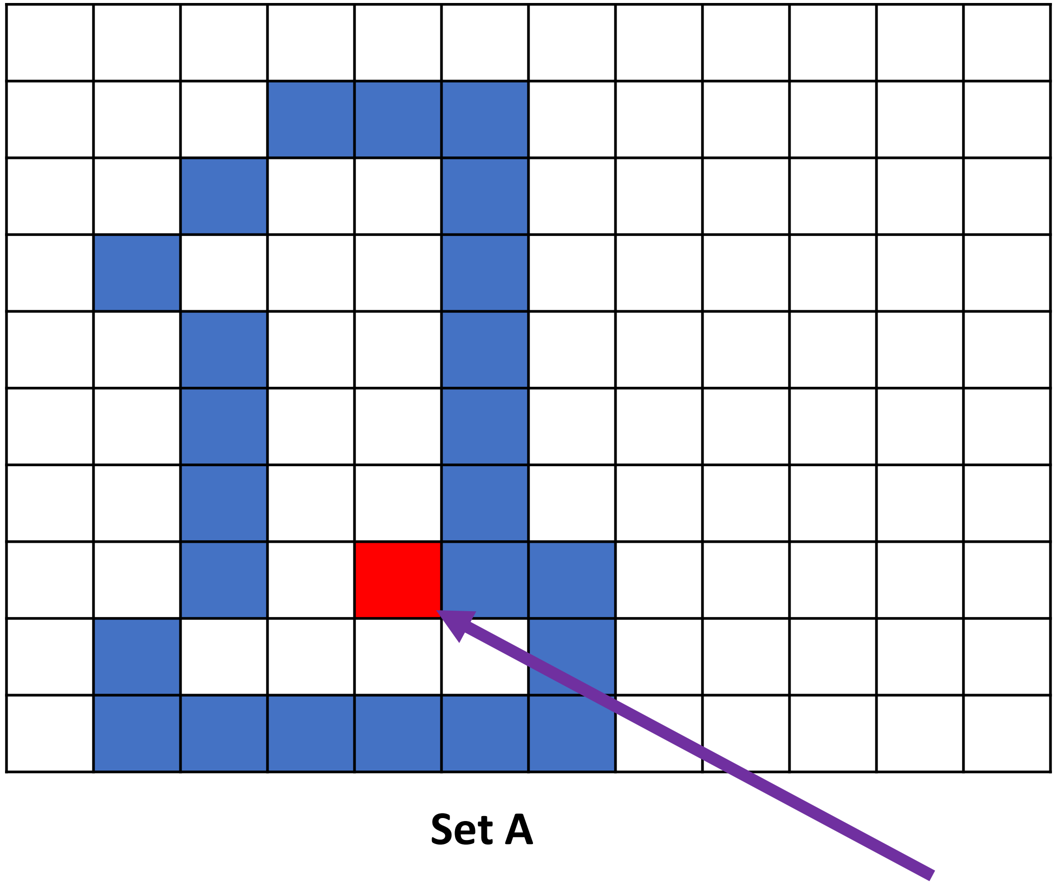


Set A



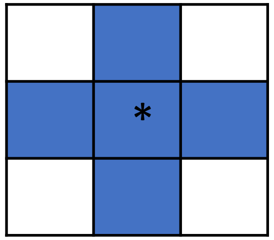
Set B

Hole Filling



Set A

We have a pixel named **P**



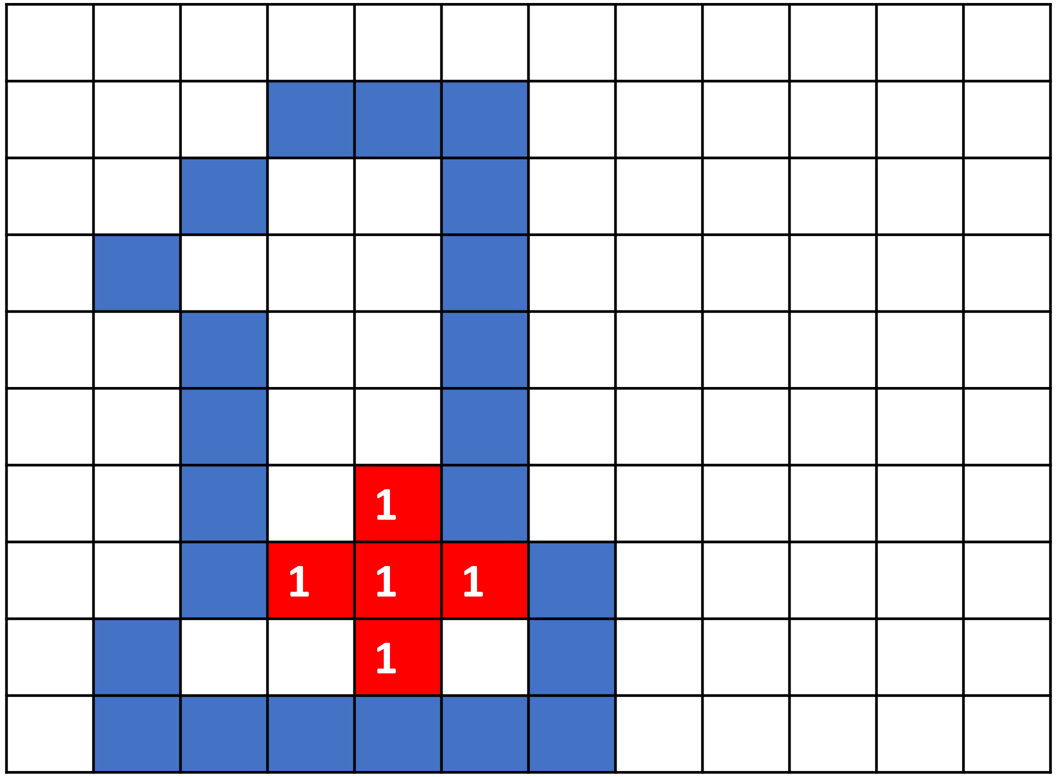
Set B

It’s an iterative process

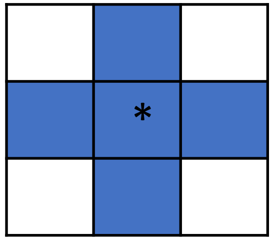
Let assume $X_0 = P$

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

Hole Filling



$X_0 \oplus B$



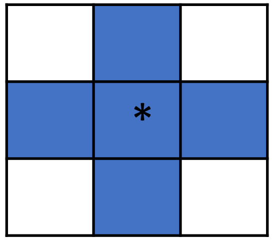
Set B

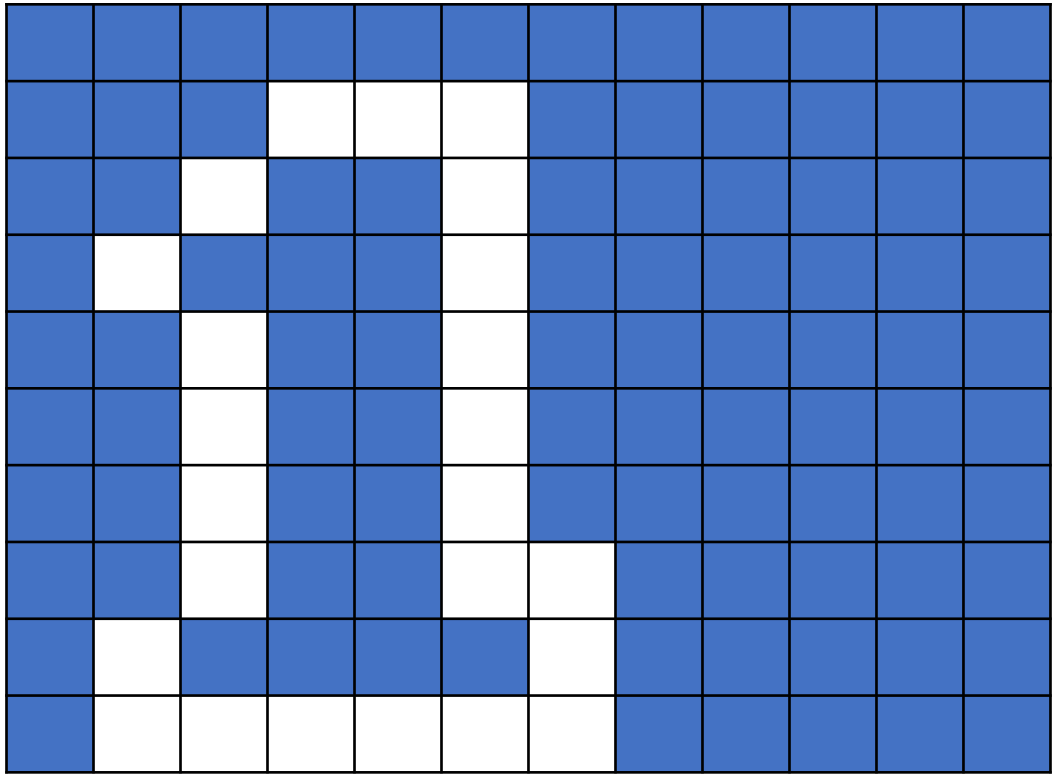
Iteration 1

$X_0 = P = 1$

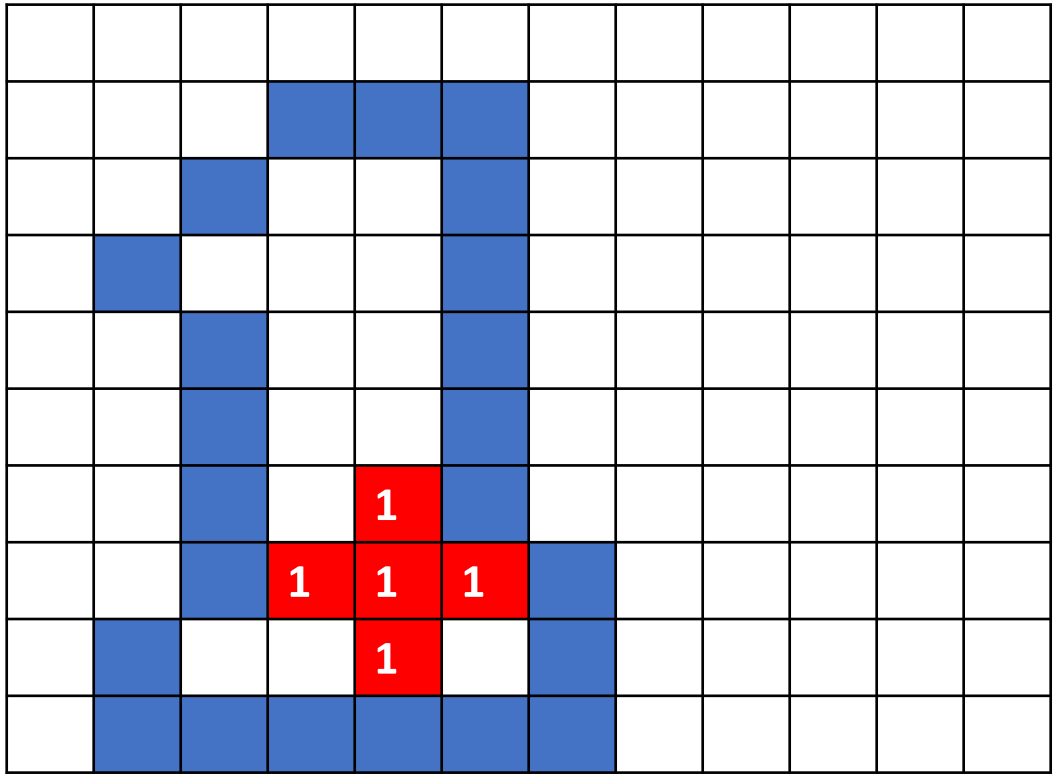
$X_1 = (X_0 \oplus B) \cap A^c$

Hole Filling





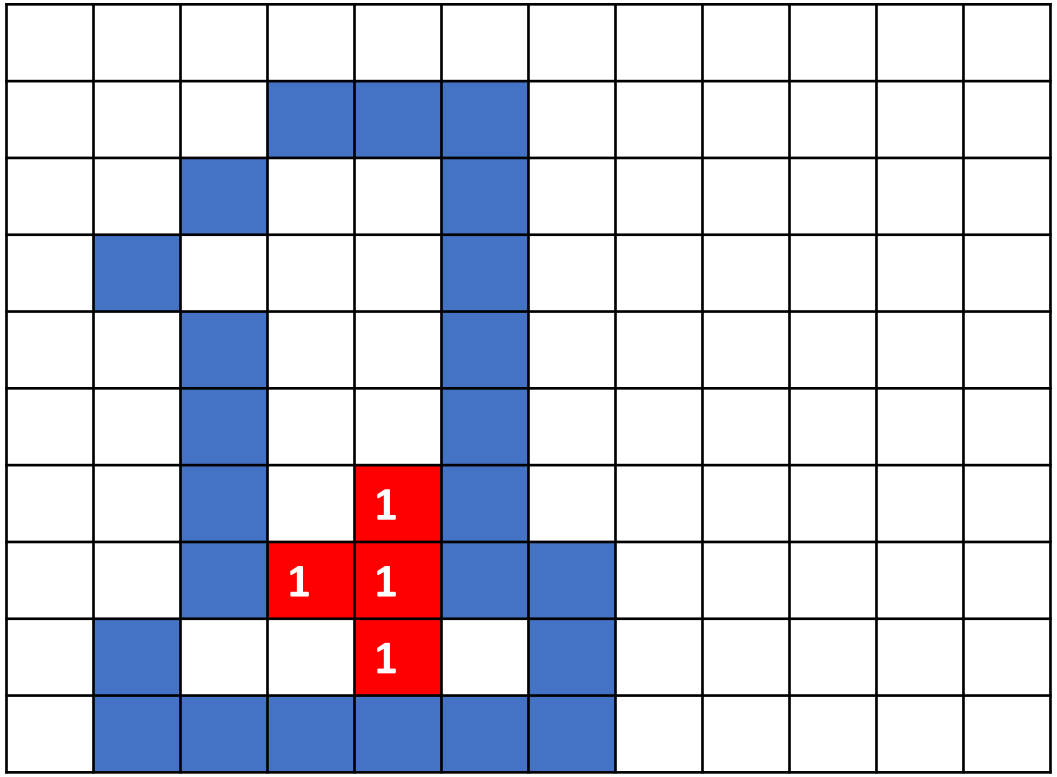
Set A^c



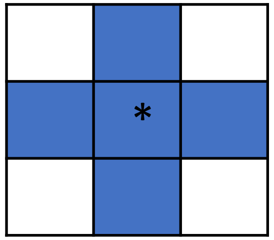
$X_0 \oplus B$

$$X_1 = (X_0 \oplus B) \cap A^c = ?$$

Hole Filling



$X_1 = (X_0 \oplus B) \cap A^c$

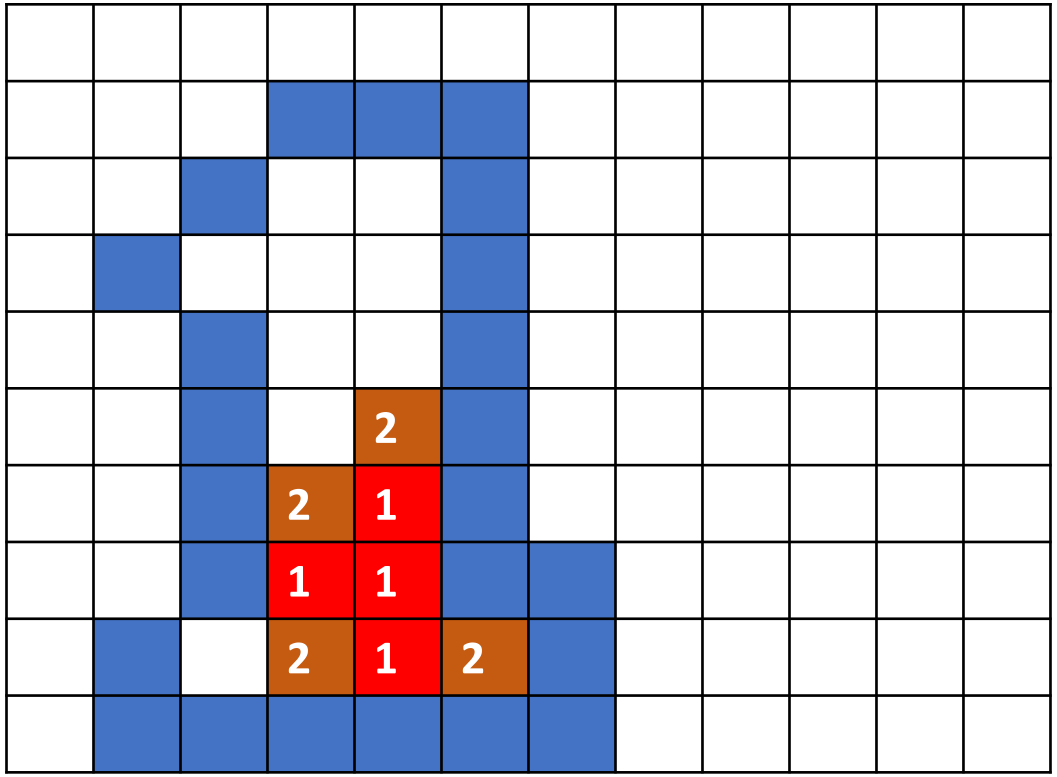


Set B

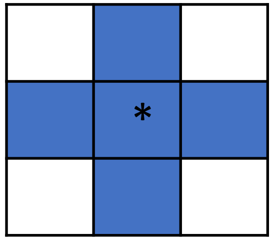
Iteration 2

$X_2 = (X_1 \oplus B) \cap A^c$

Hole Filling



$X_2 = (X_1 \oplus B) \cap A^c$

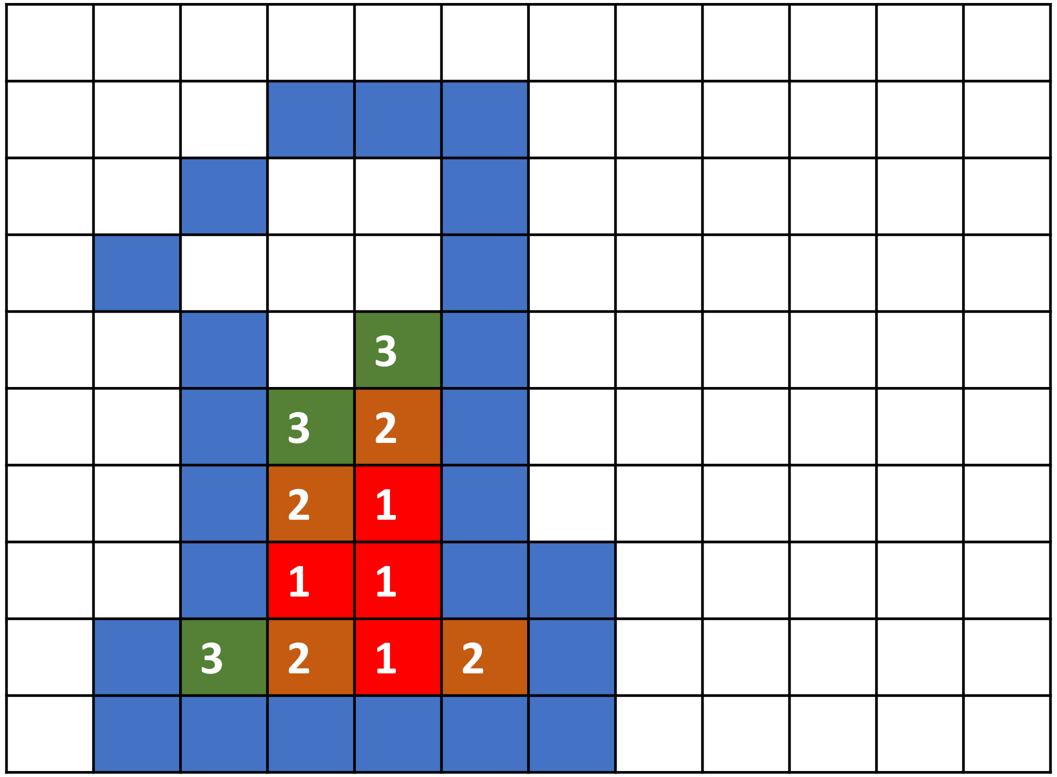


Set B

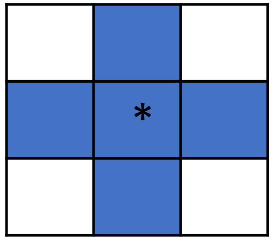
Iteration 3

$X_3 = (X_2 \oplus B) \cap A^c$

Hole Filling



$X_3 = (X_2 \oplus B) \cap A^c$

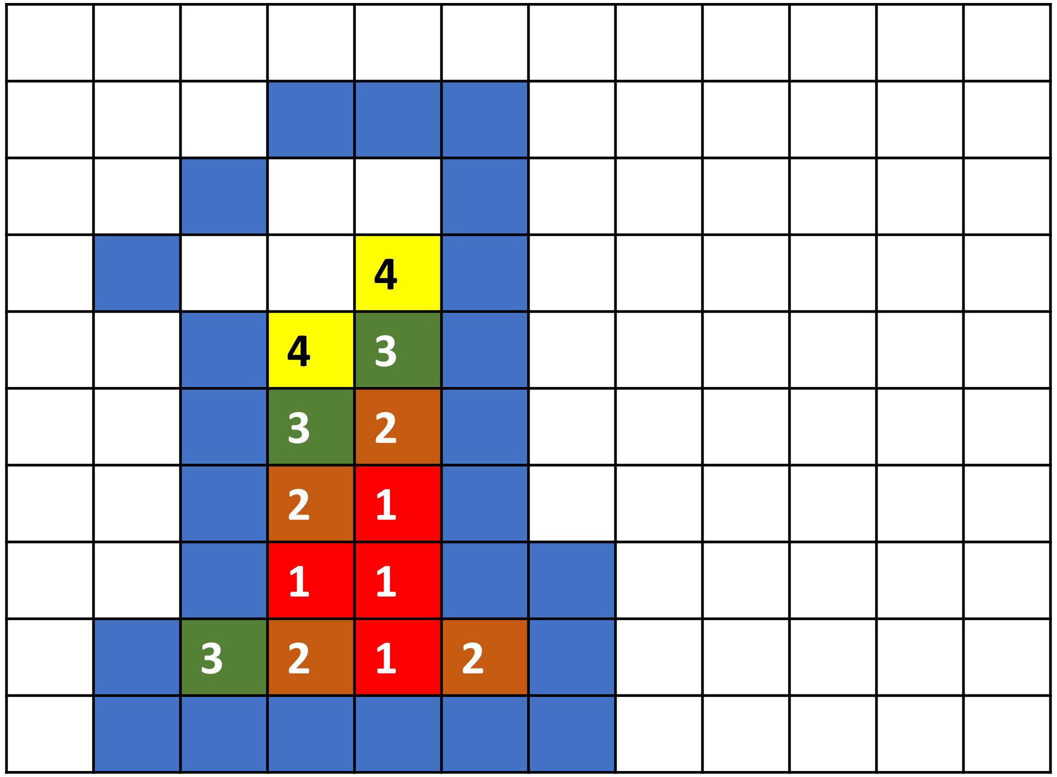


Set B

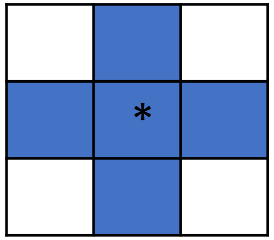
Iteration 4

$X_4 = (X_3 \oplus B) \cap A^c$

Hole Filling



$X_4 = (X_3 \oplus B) \cap A^c$

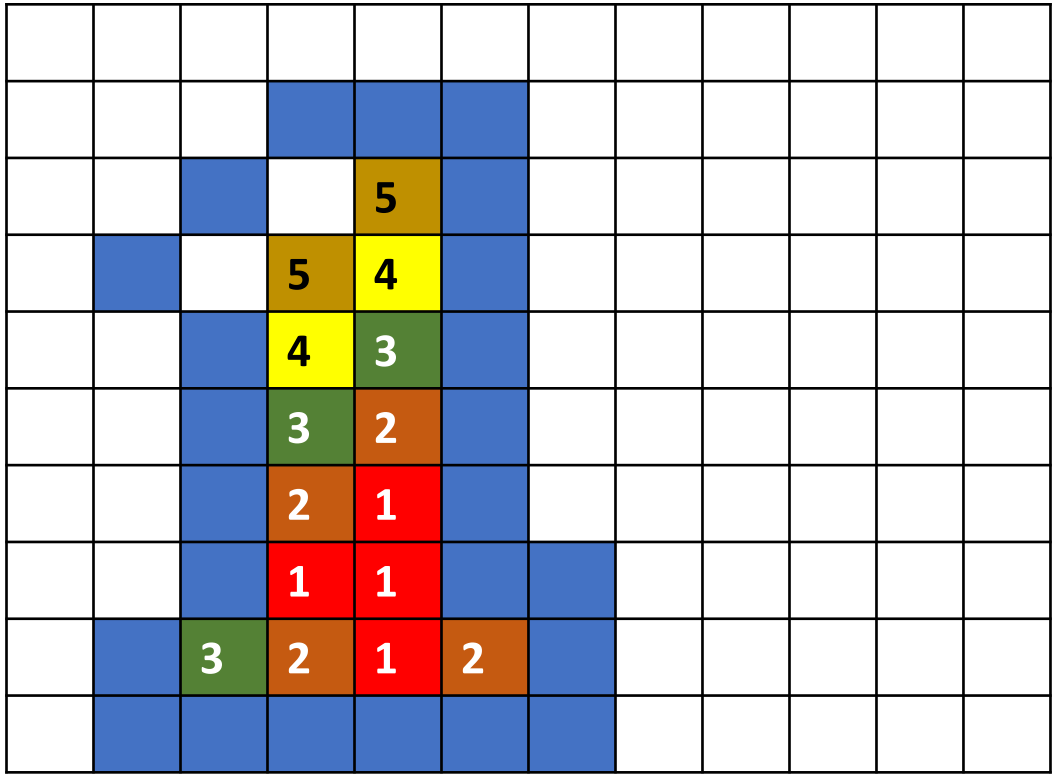


Set B

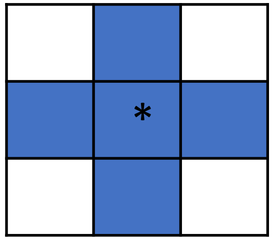
Iteration 5

$X_5 = (X_4 \oplus B) \cap A^c$

Hole Filling



$X_5 = (X_4 \oplus B) \cap A^c$

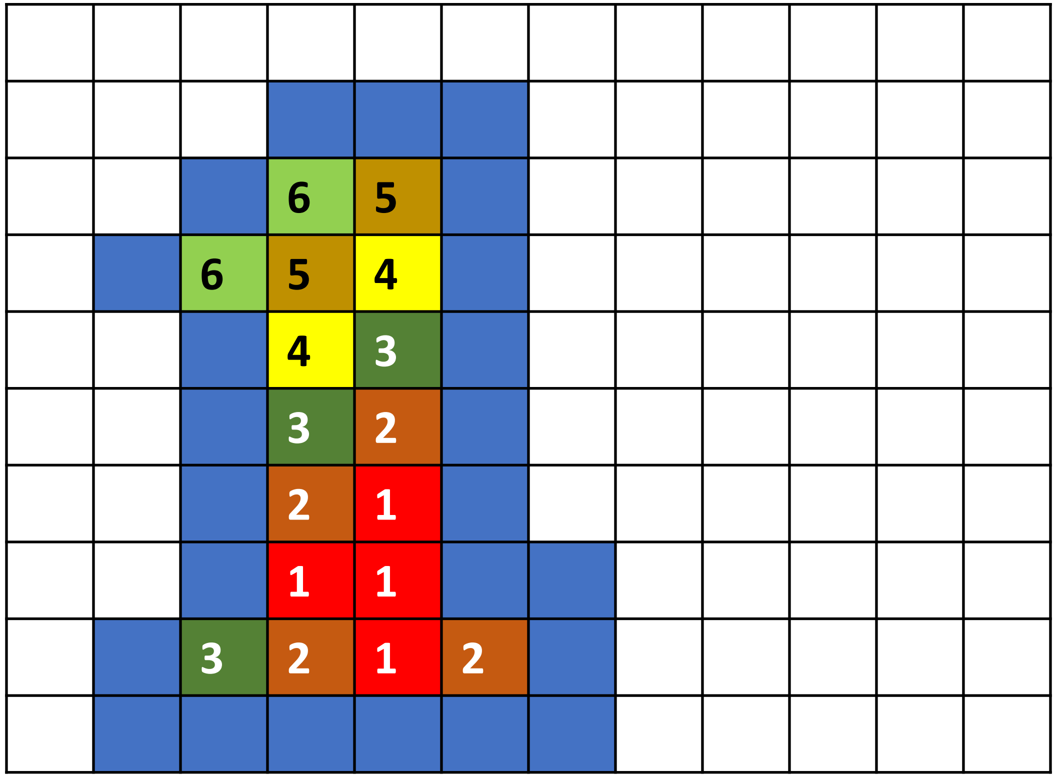


Set B

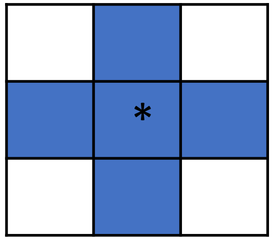
Iteration 6

$X_6 = (X_5 \oplus B) \cap A^c$

Hole Filling



$X_6 = (X_5 \oplus B) \cap A^c$

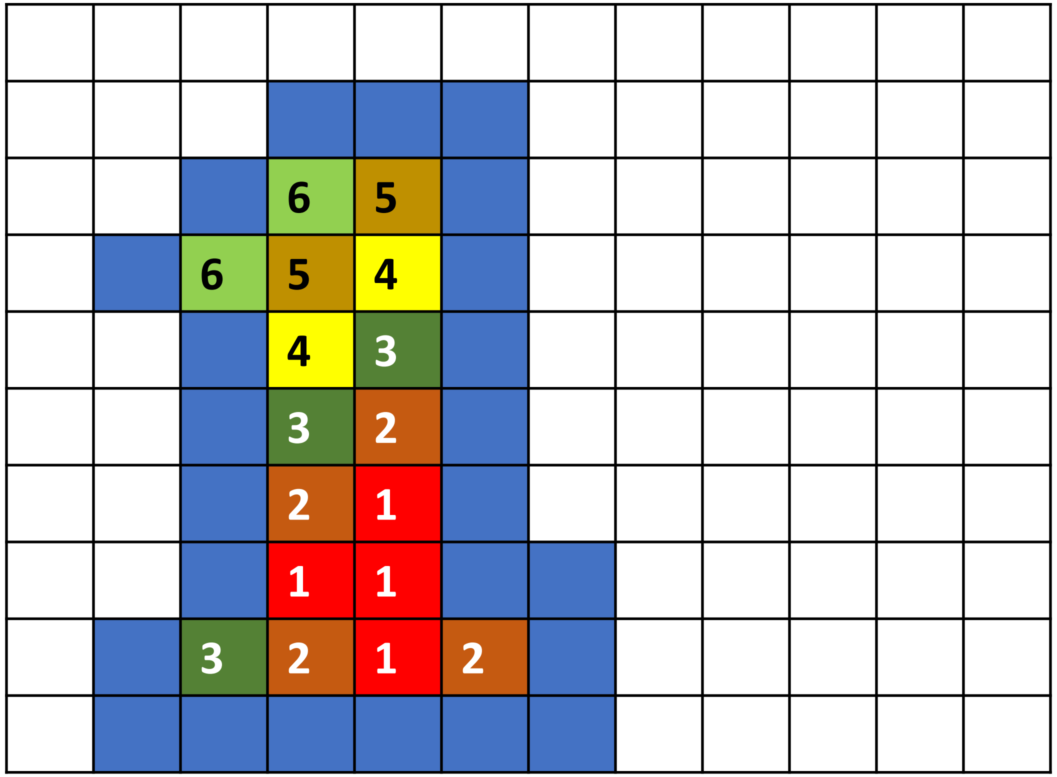


Set B

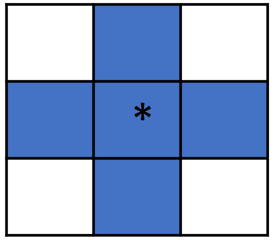
Iteration 7

$X_7 = (X_6 \oplus B) \cap A^c$

Hole Filling



$X_7 = (X_6 \oplus B) \cap A^c$



Set B

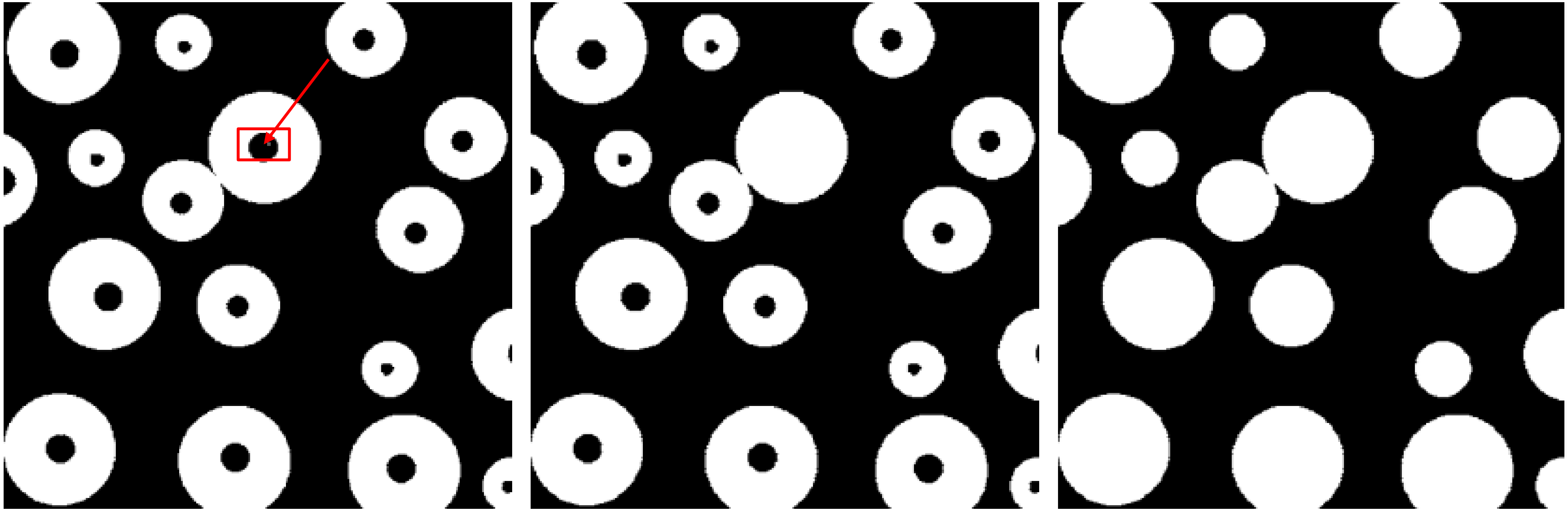
Process terminate

$X_7 = X_6$

$X_k = X_{k-1}$

Final Set = $X_k \cup A$

Hole Filling



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.

The Hit-or-Miss Transform

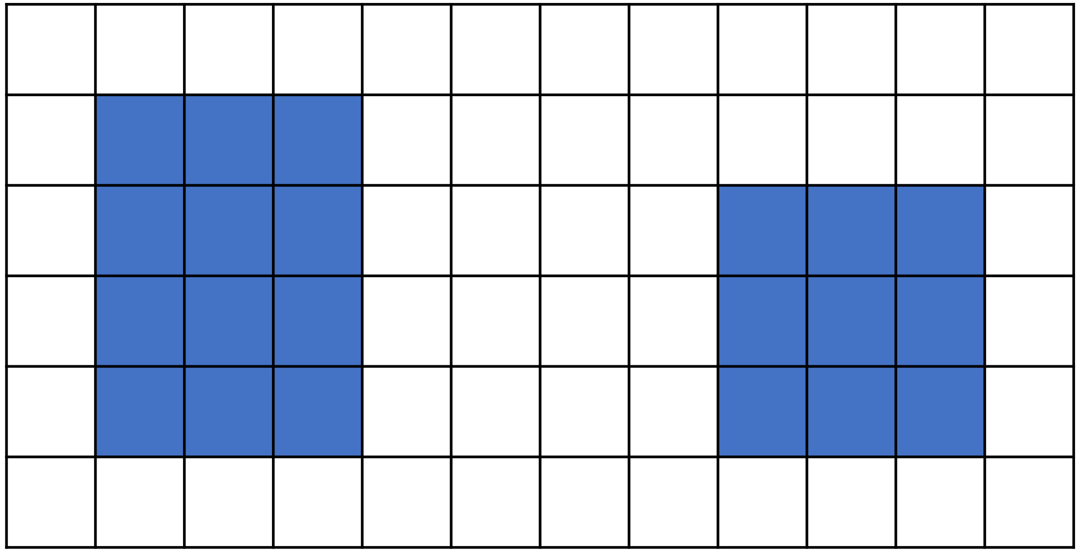
- **A basic tools to detect object with given shape and size**

Generalized equation

$B = (B_1, B_2)$

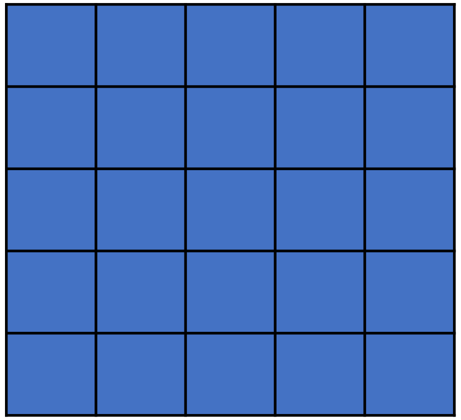
$A \circledast B = (A \ominus \mathbf{B_1}) \cap (\mathbf{A^c} \ominus \mathbf{B_2})$

Hit-or-Miss Transformation

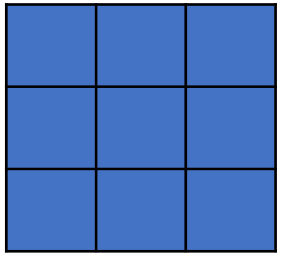


Set A

W

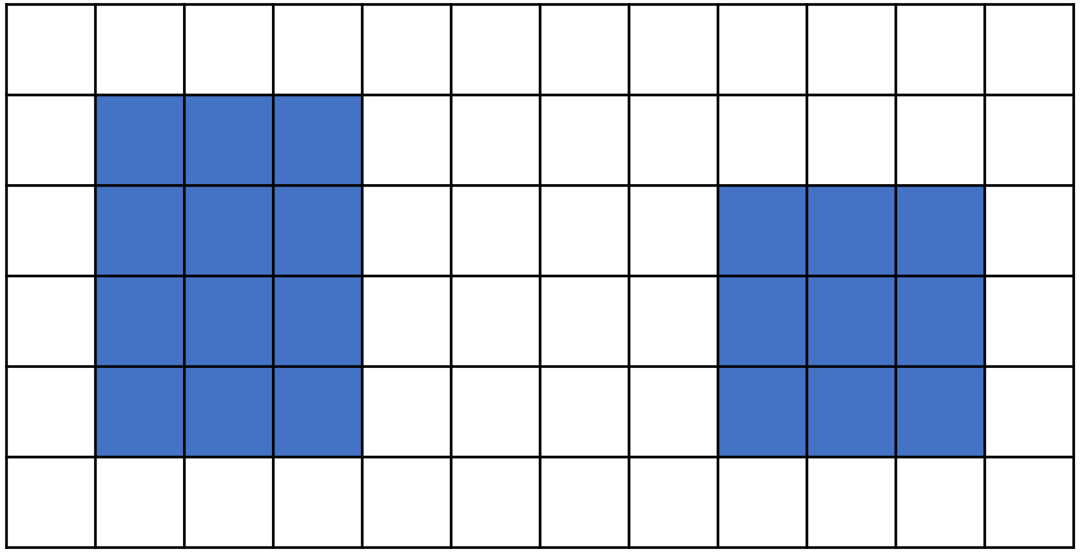


X

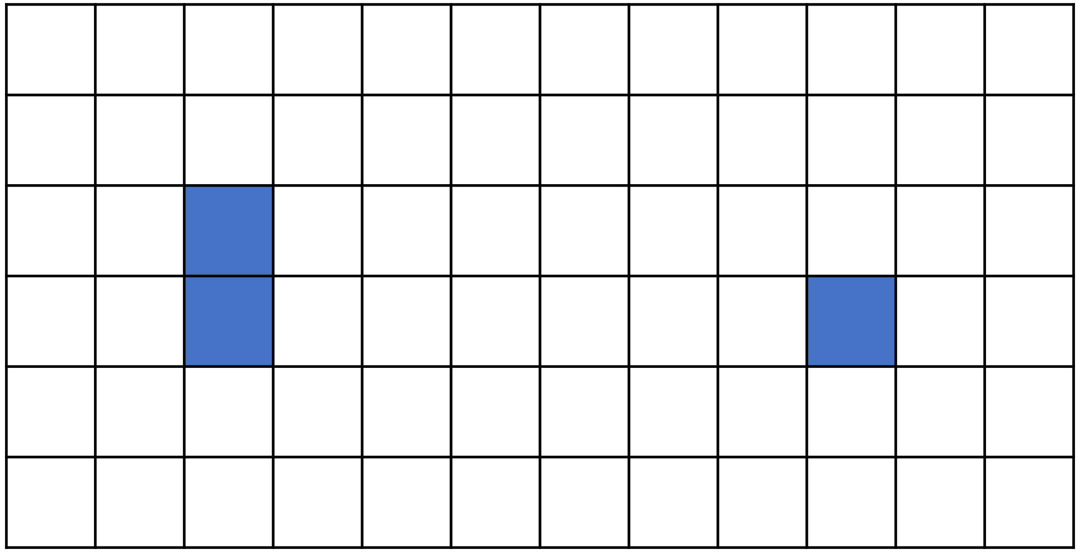


Structure element

Hit-or-Miss Transformation

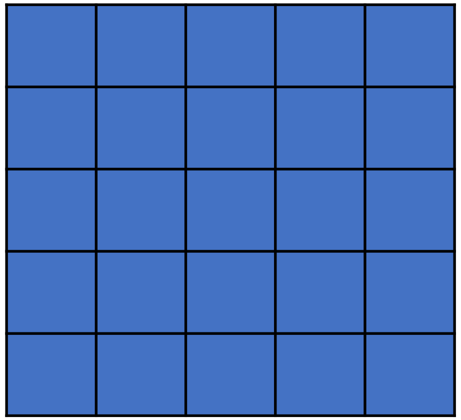


Set A

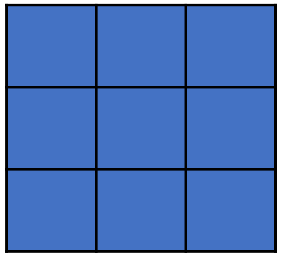


A Ө X

W

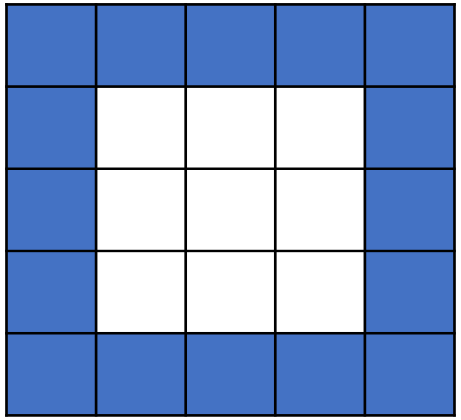


X

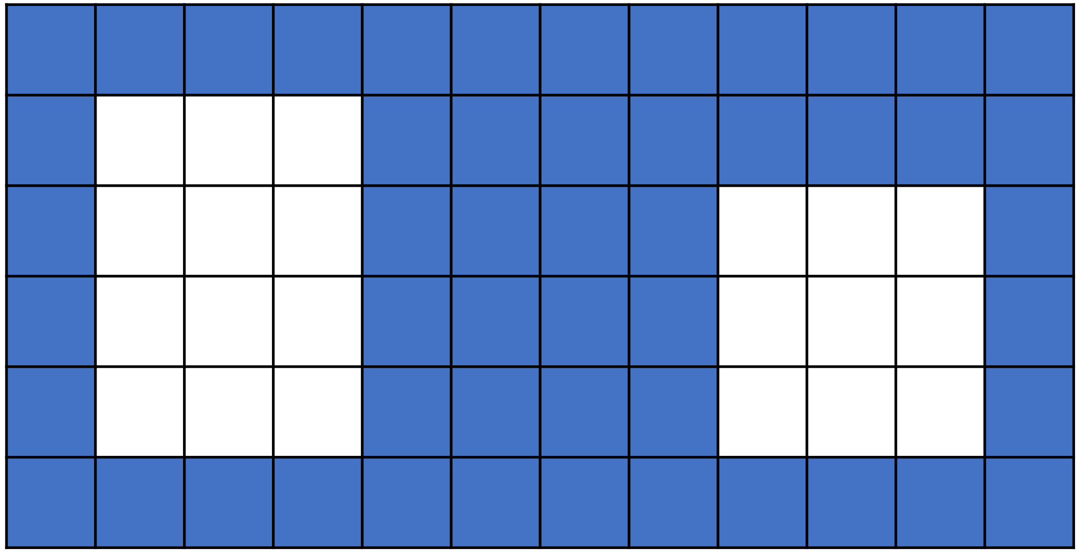


Structure element

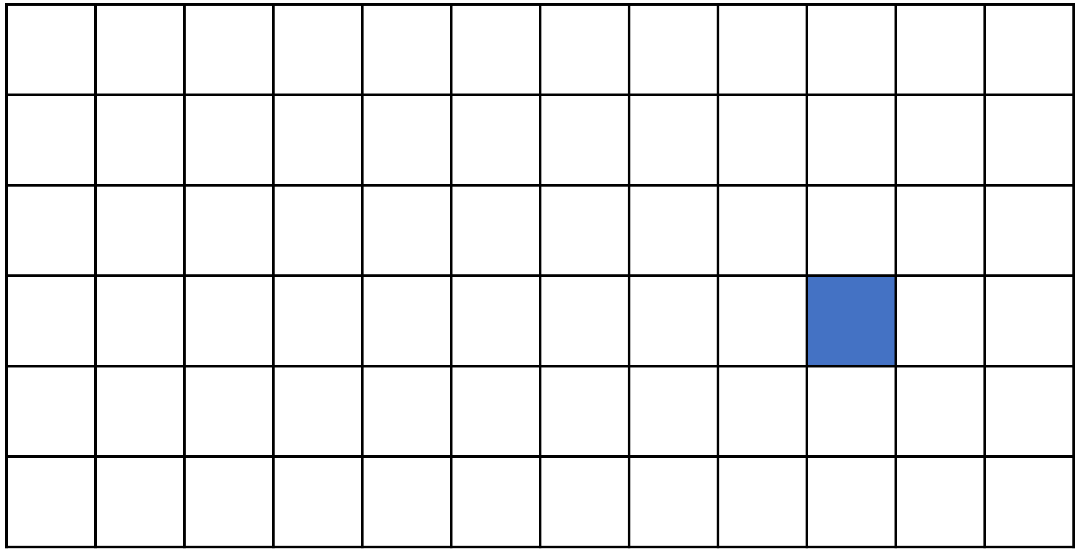
W-X



Hit-or-Miss Transformation

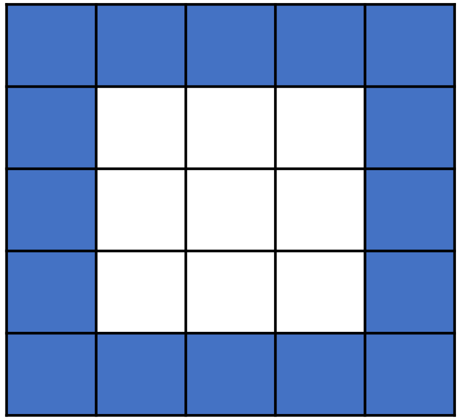


A complement

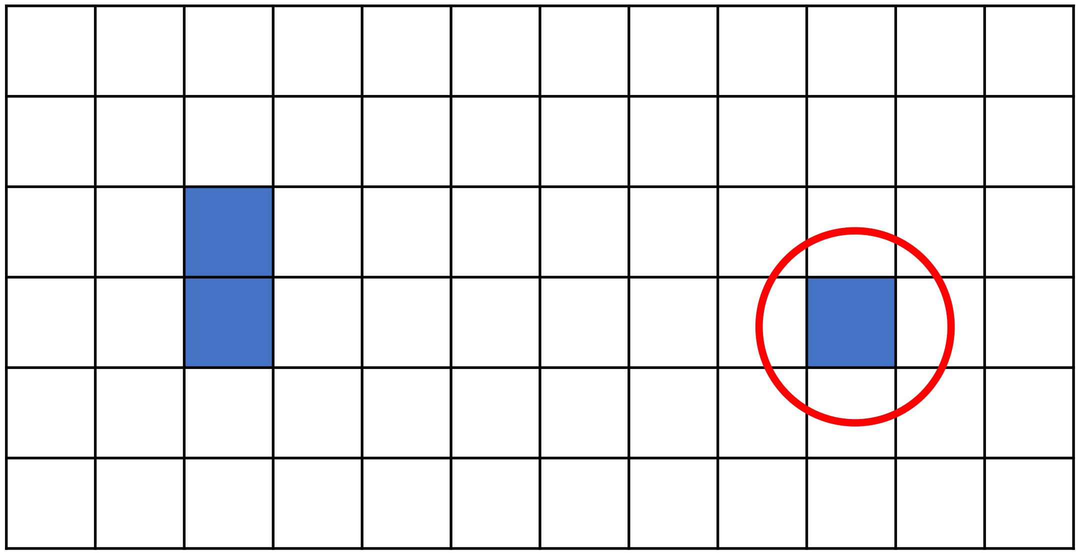


$A^C \ominus (W-X)$

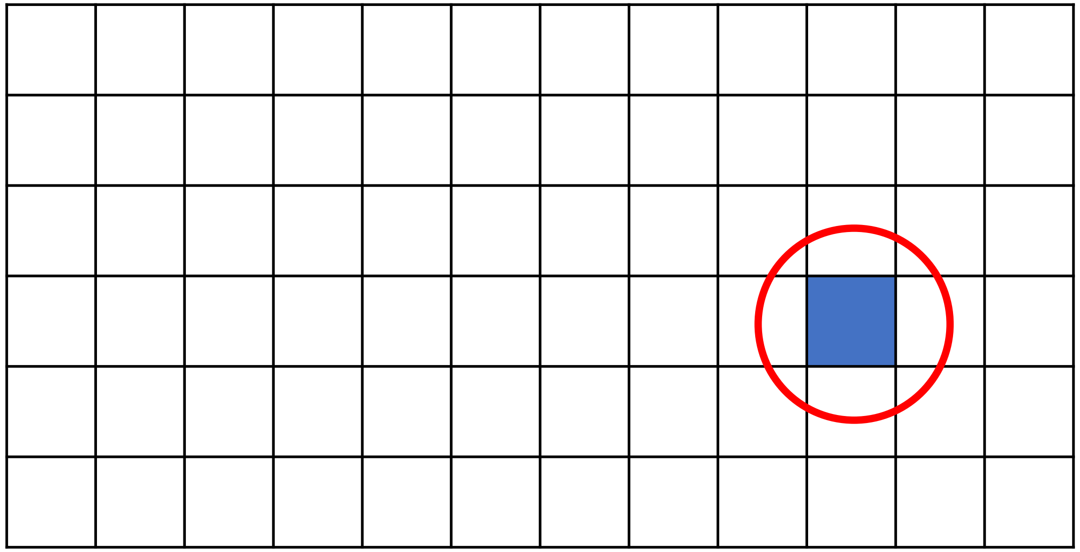
W-X



Hit-or-Miss Transformation



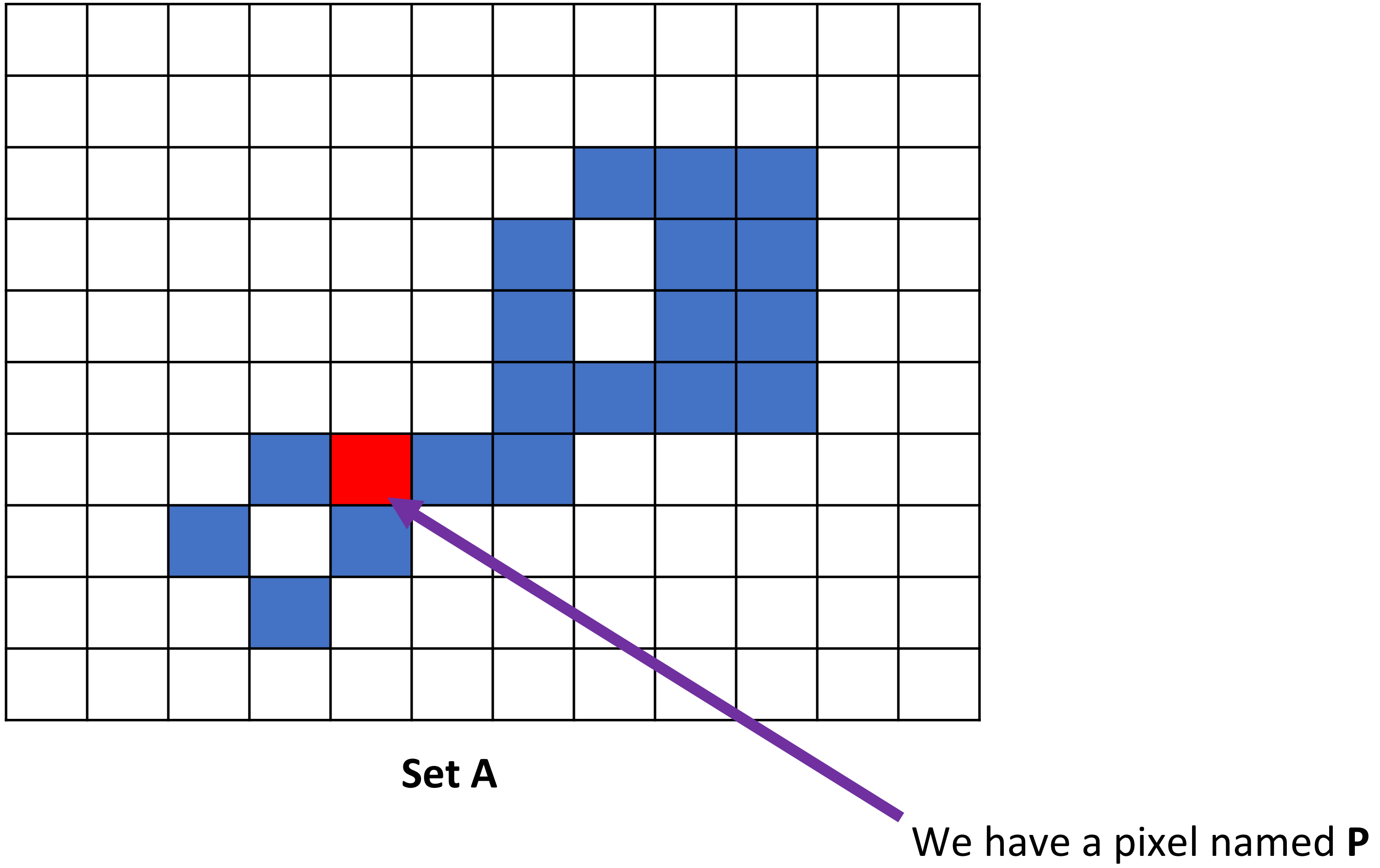
A ⊖ X



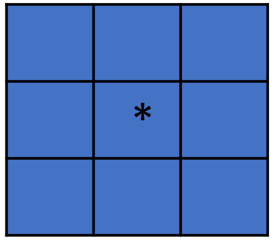
$A^C ⊖ (W-X)$

$$(A \ominus X) \cap (A^C \ominus (W-X)) = ?$$

Extraction of Connected Components



Set A



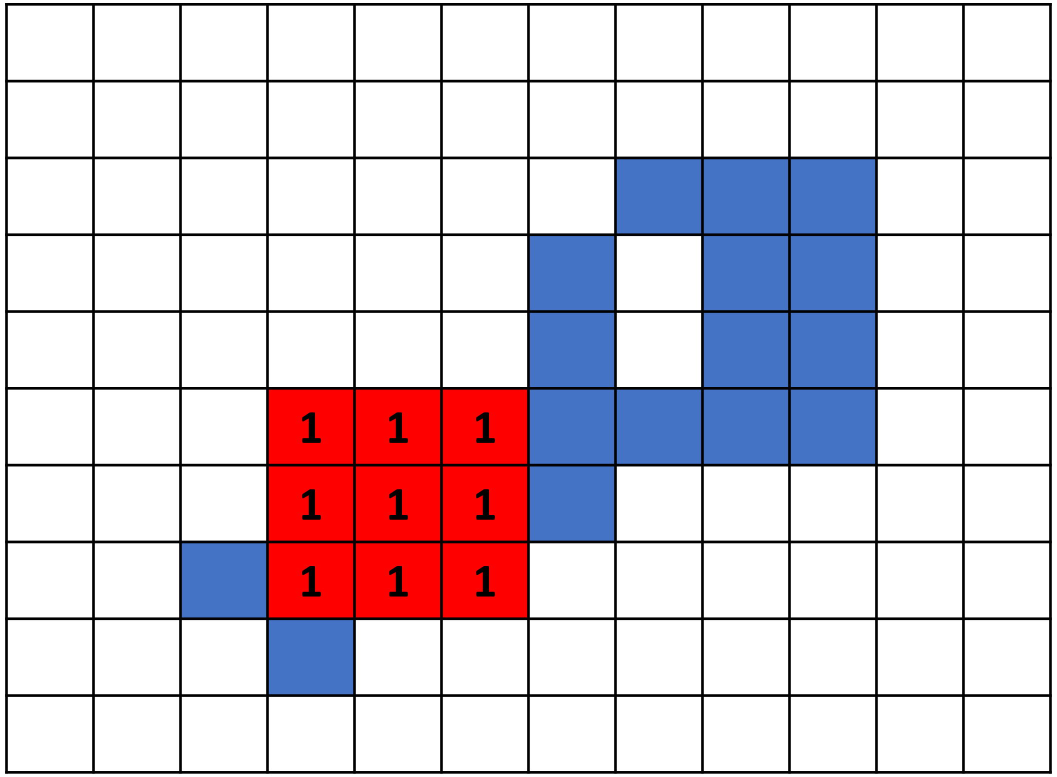
Set B

It's an iterative process

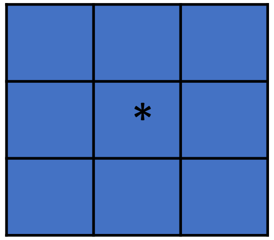
Let assume $X_0 = P$

$X_k = (X_{k-1} \oplus B) \cap A$

Extraction of Connected Components



$X_0 \oplus B$



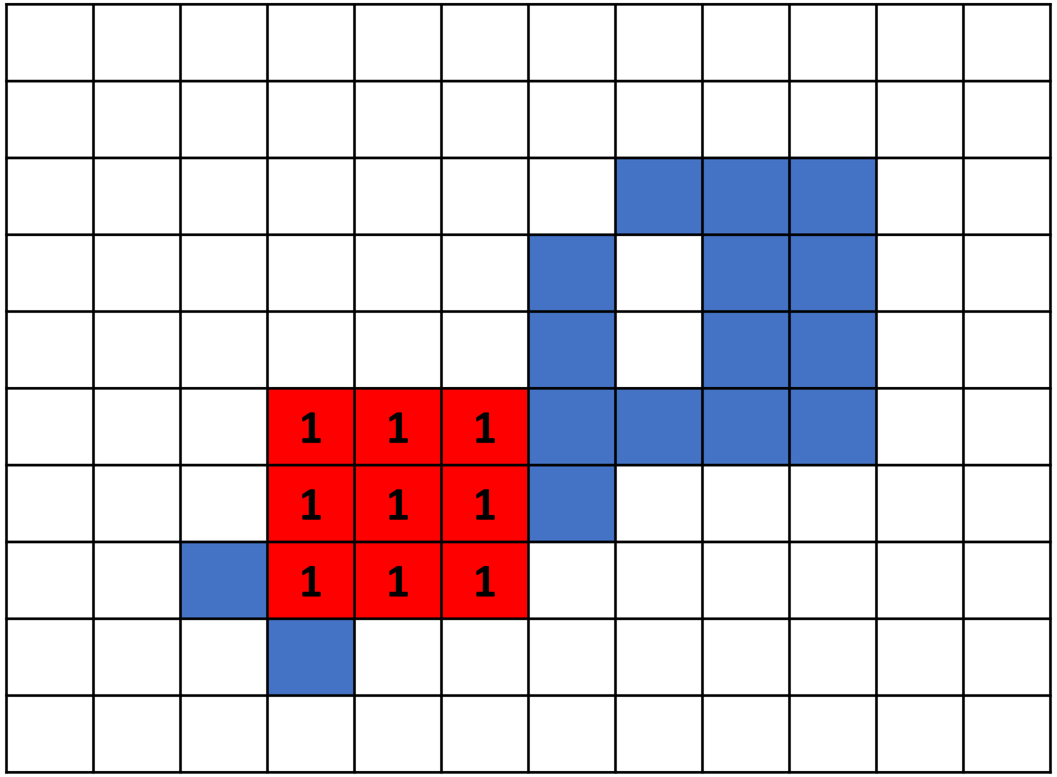
Set B

Iteration 1

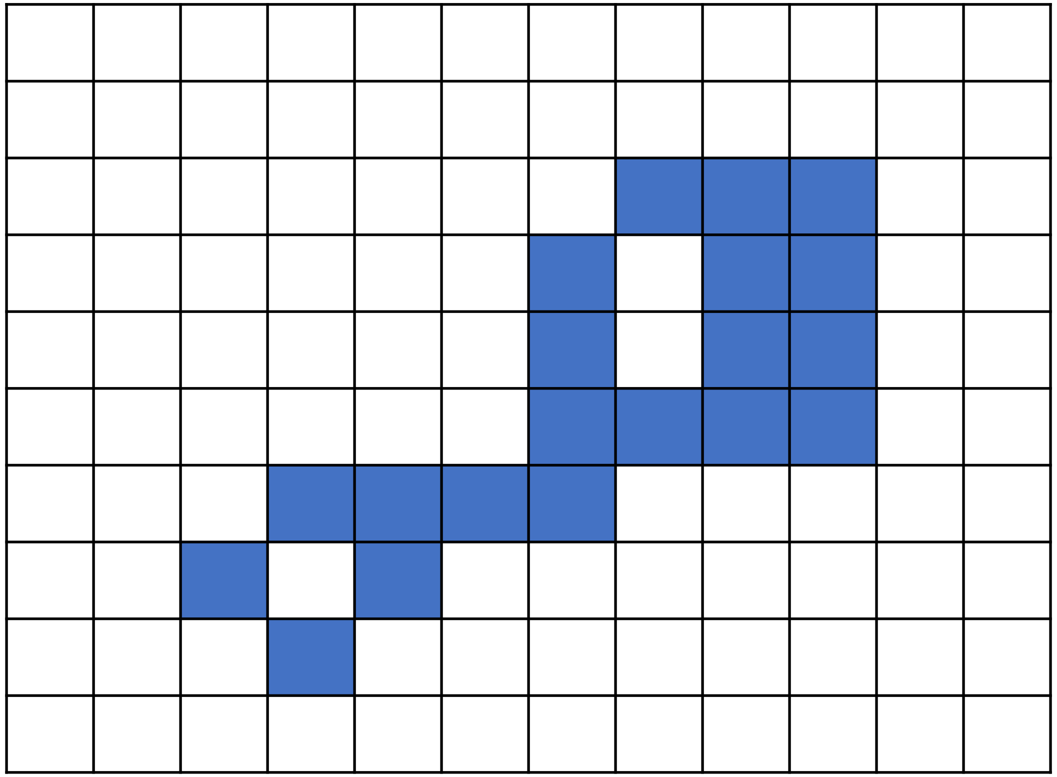
$$X_0 = P$$

$$X_1 = (X_0 \oplus B) \cap A$$

Extraction of Connected Components



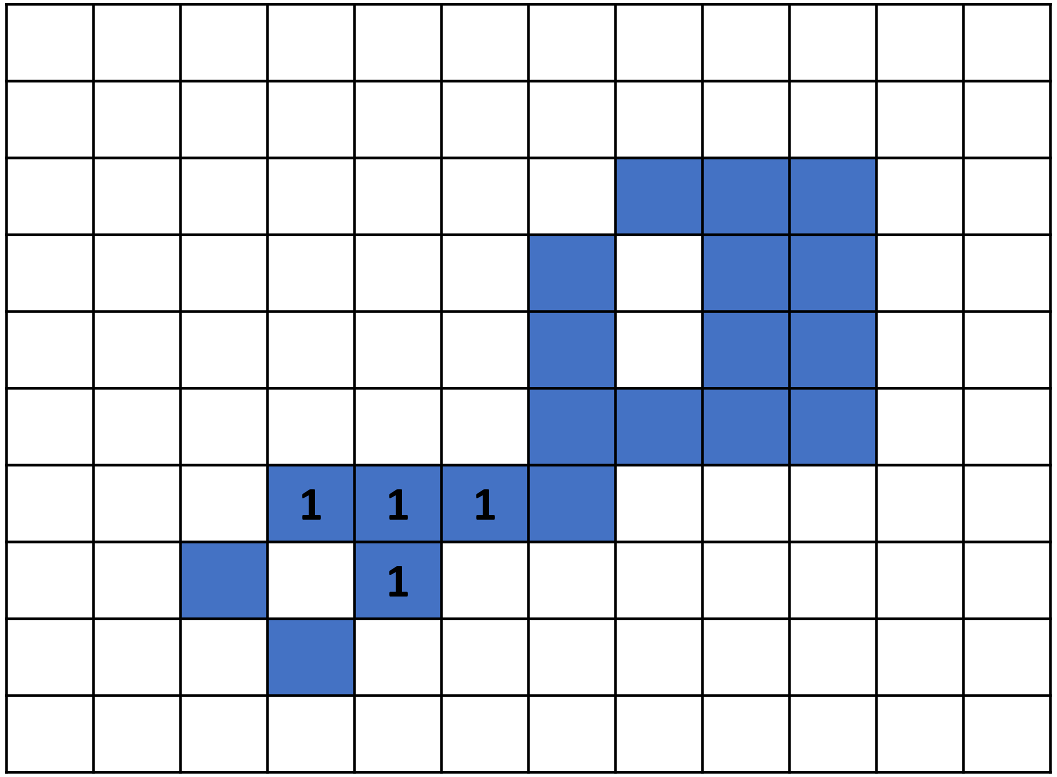
$X_0 \oplus B$



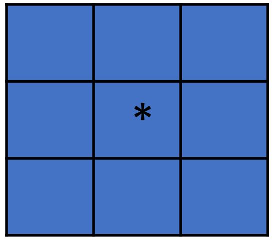
Set A

$$X_1 = (X_0 \oplus B) \cap A = ?$$

Extraction of Connected Components



$X_1 = (X_0 \oplus B) \cap A$

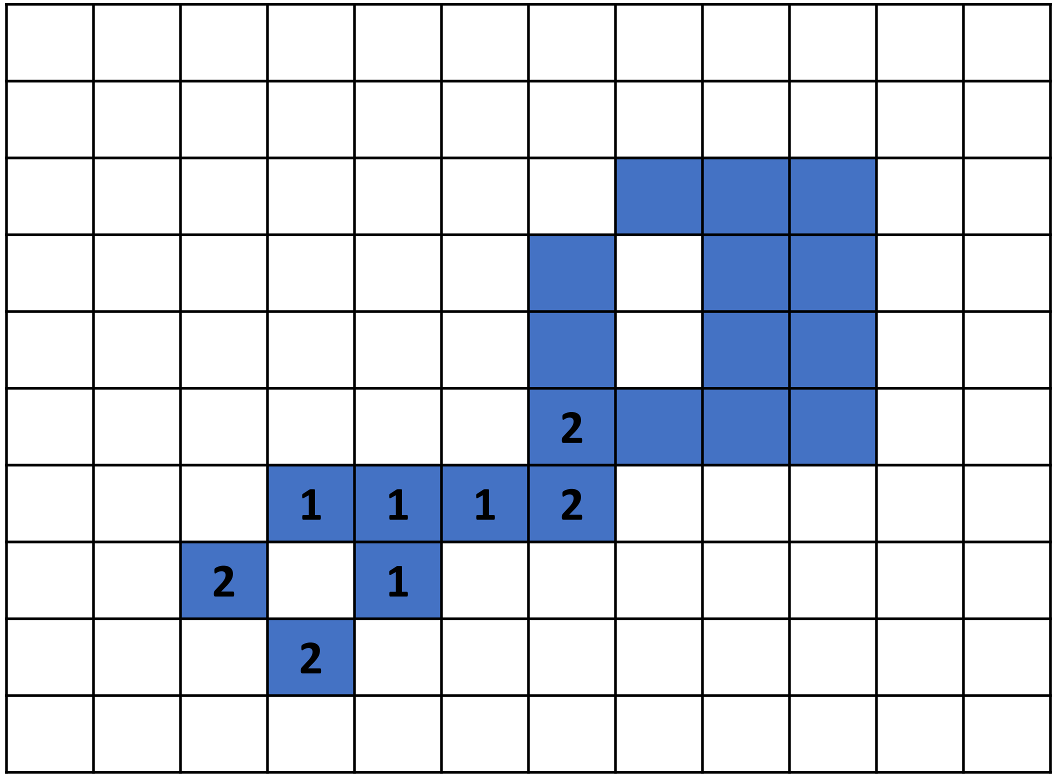


Set B

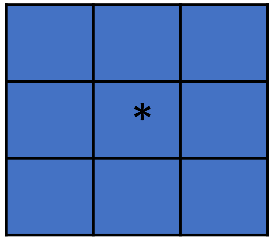
Iteration 2

$X_2 = (X_1 \oplus B) \cap A$

Extraction of Connected Components



$X_2 = (X_1 \oplus B) \cap A$

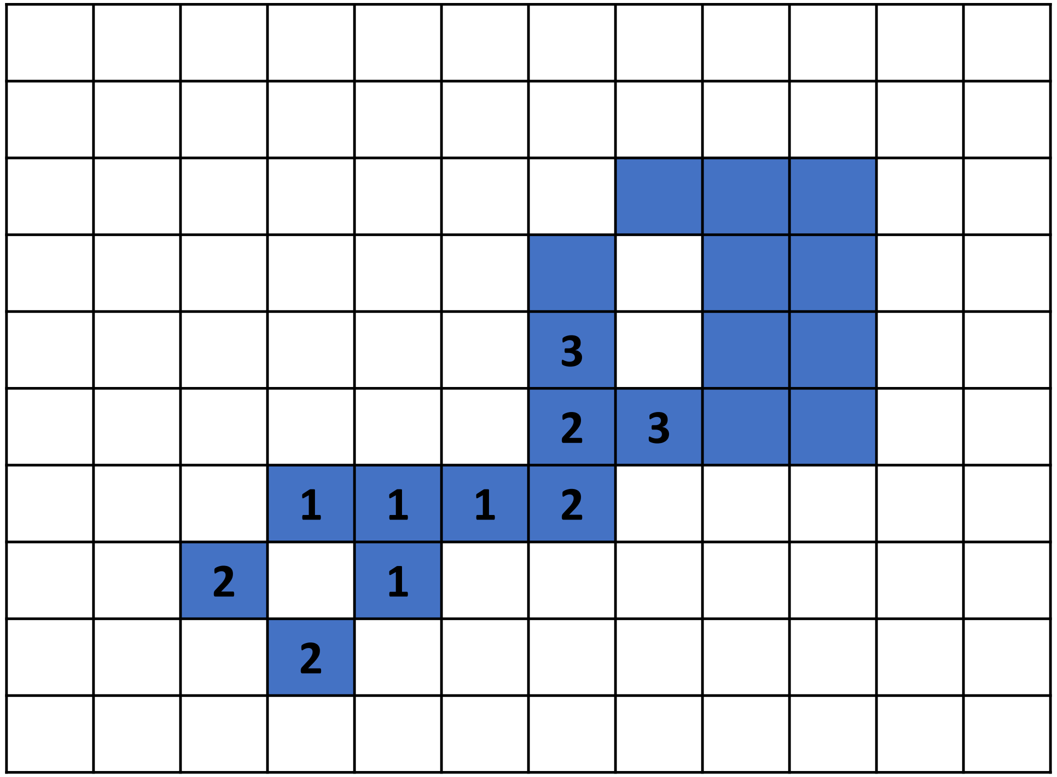


Set B

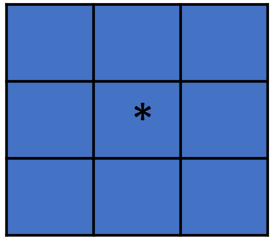
Iteration 3

$X_3 = (X_2 \oplus B) \cap A$

Extraction of Connected Components



$X_3 = (X_2 \oplus B) \cap A$

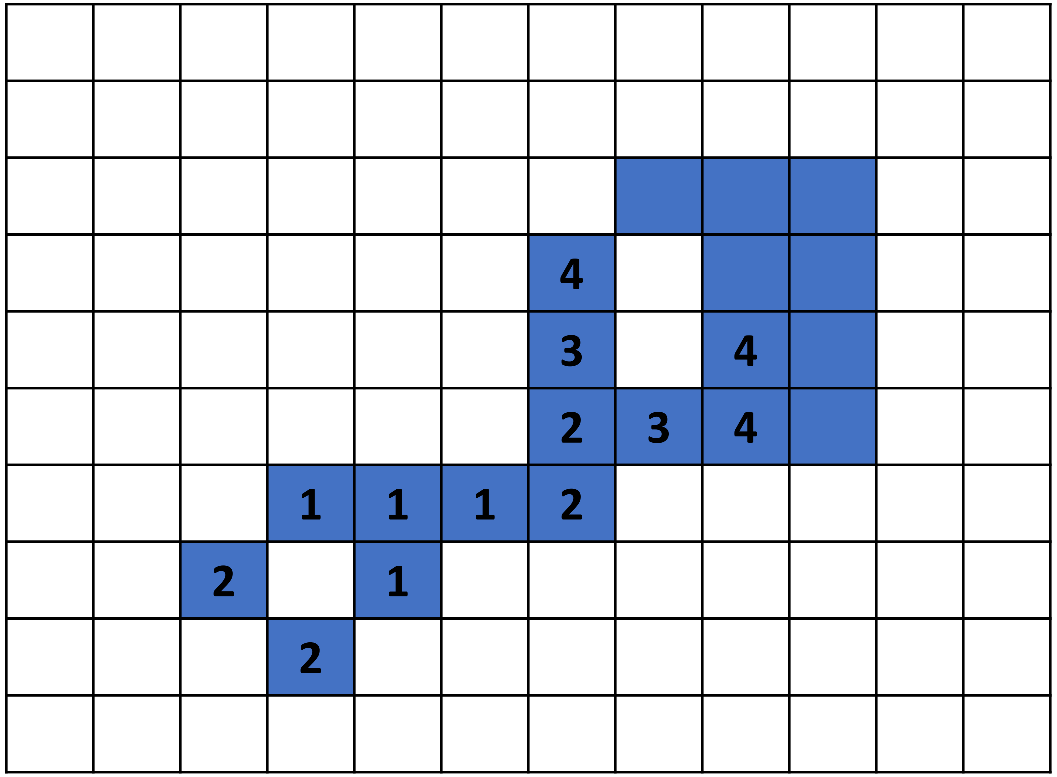


Set B

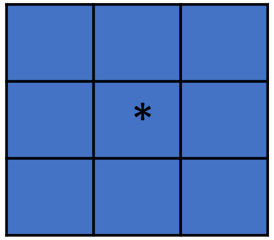
Iteration 4

$X_4 = (X_3 \oplus B) \cap A$

Extraction of Connected Components



$X_4 = (X_3 \oplus B) \cap A$

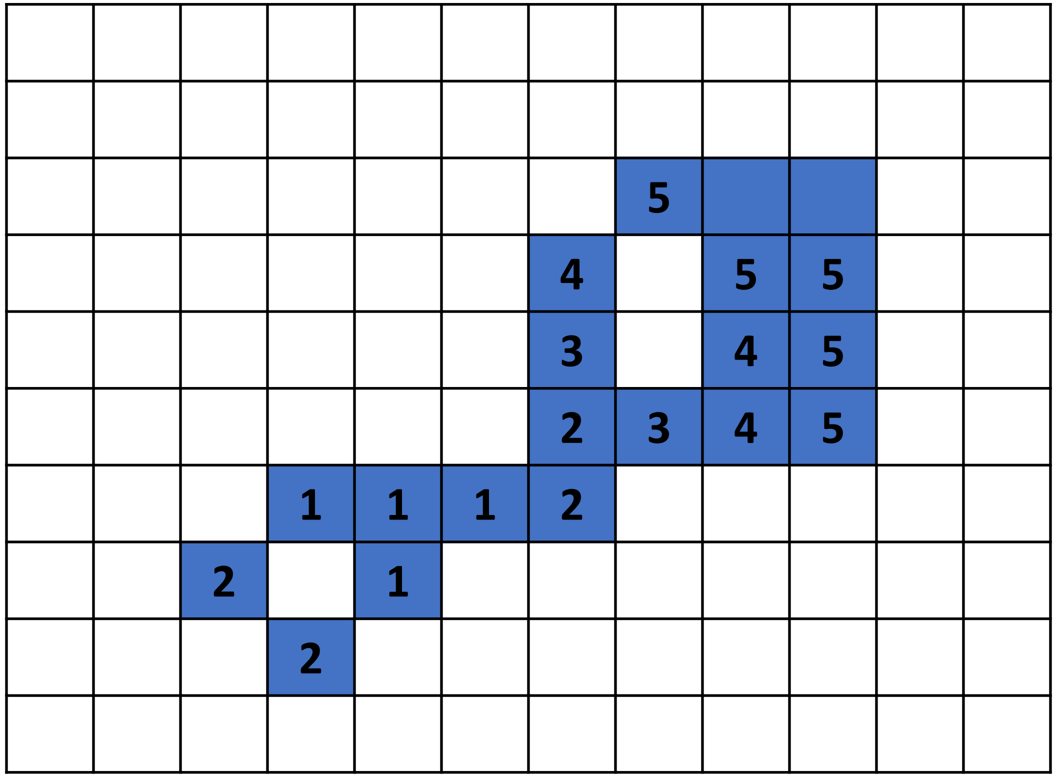


Set B

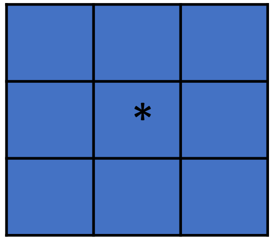
Iteration 5

$X_5 = (X_4 \oplus B) \cap A$

Extraction of Connected Components



$X_5 = (X_4 \oplus B) \cap A$

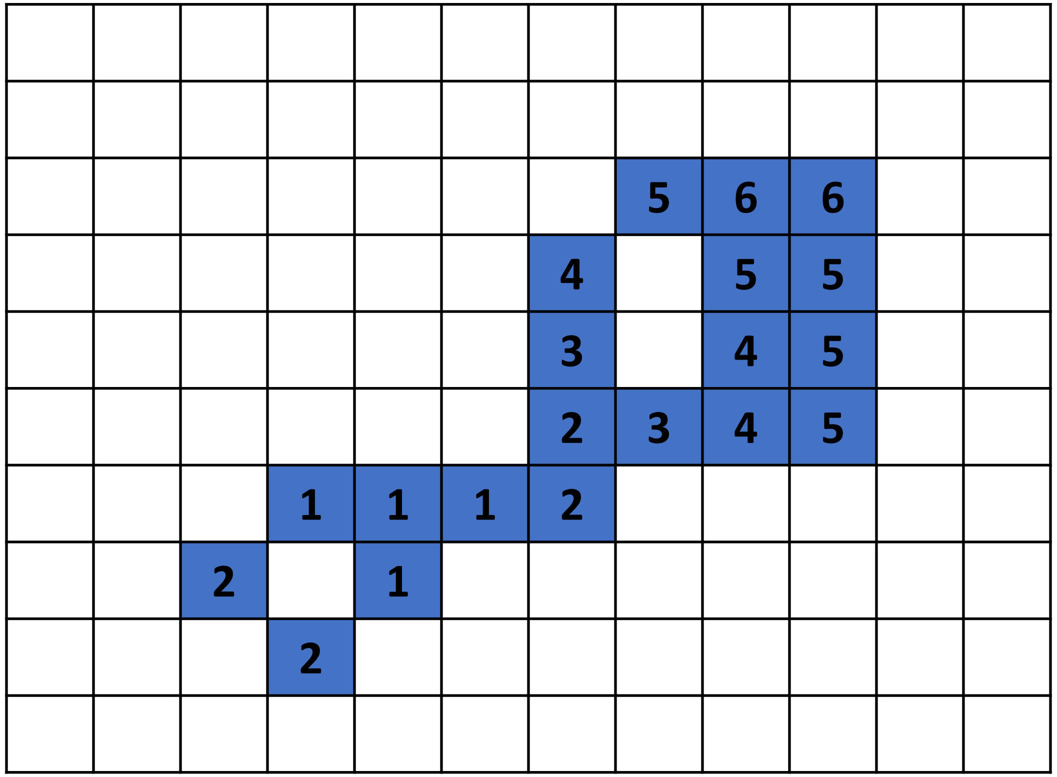


Set B

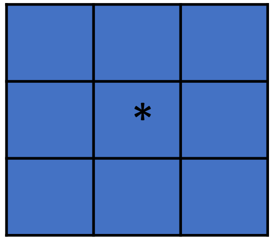
Iteration 6

$X_6 = (X_5 \oplus B) \cap A$

Extraction of Connected Components



$X_6 = (X_5 \oplus B) \cap A$

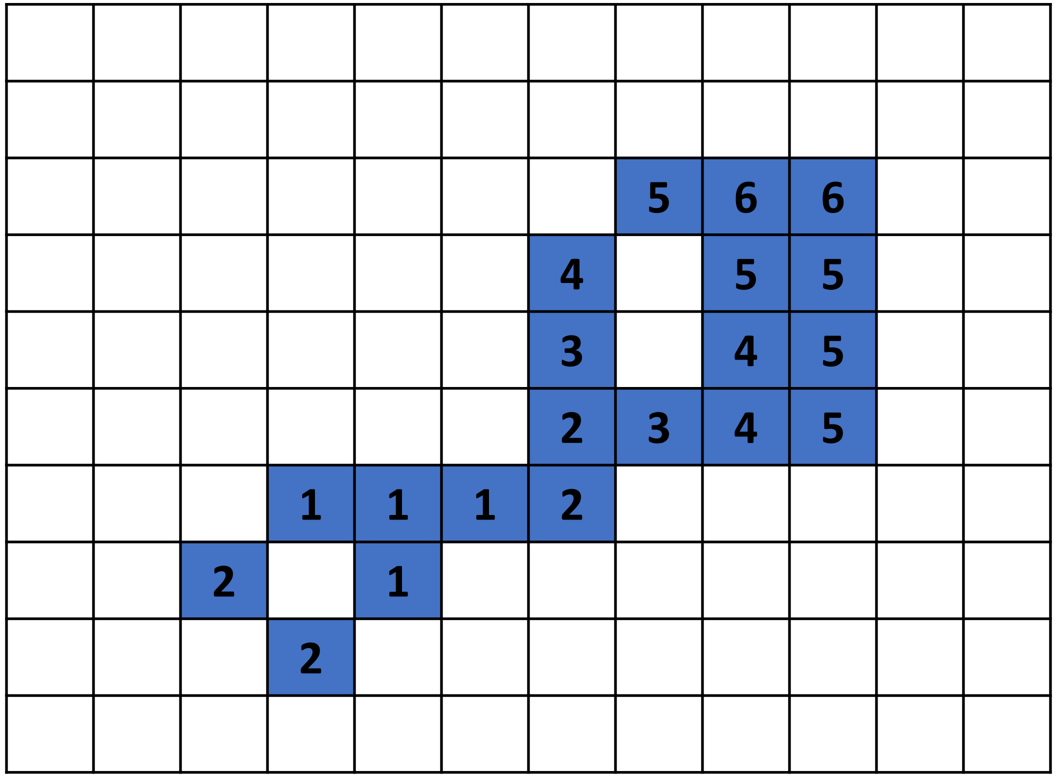


Set B

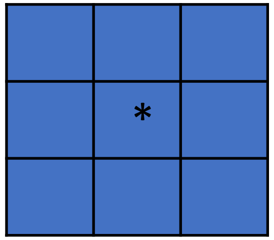
Iteration 7

$X_7 = (X_6 \oplus B) \cap A$

Extraction of Connected Components



$X_7 = (X_6 \oplus B) \cap A$

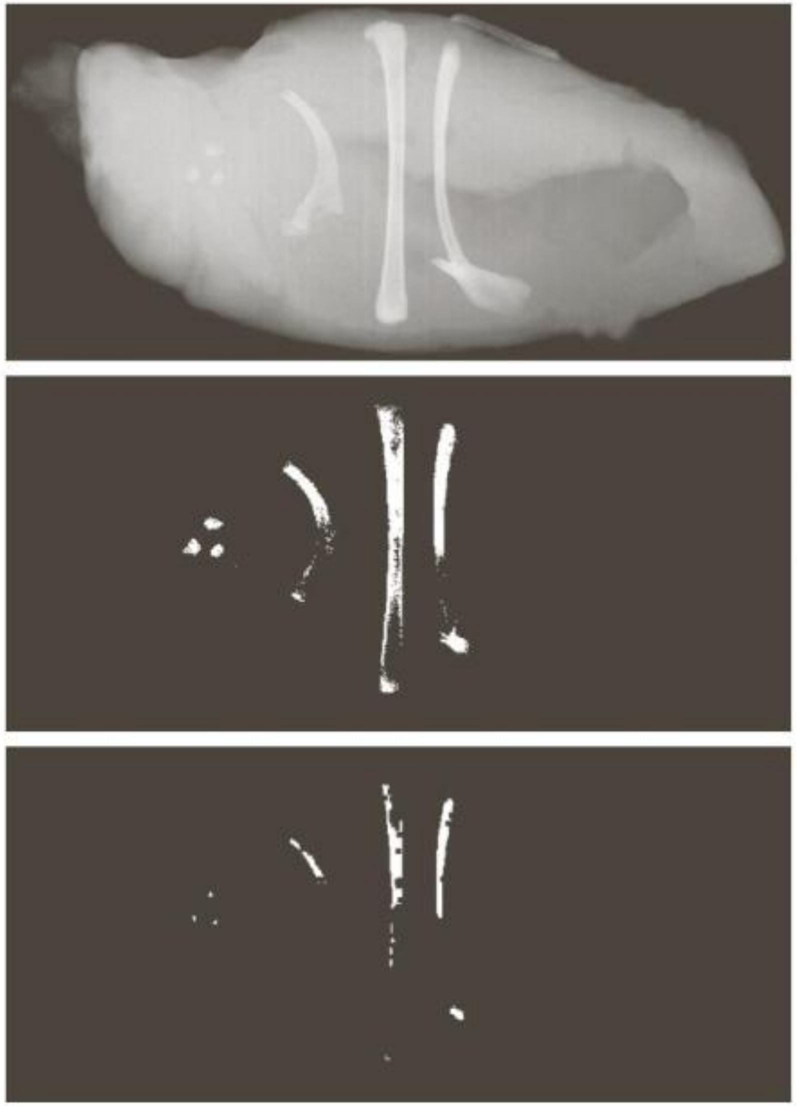


Set B

Process Terminate

$X_7 = X_6$

Extraction of Connected Components



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1s. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Skeletons

- The skeleton of A can be expressed in term of erosion and openings

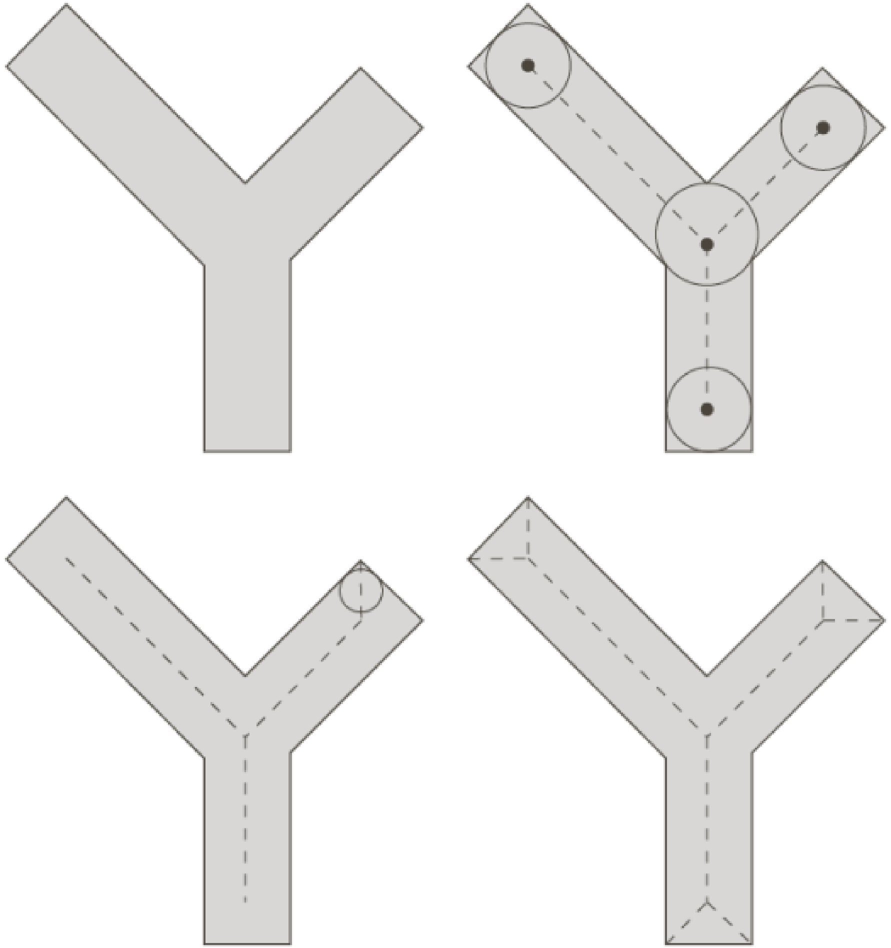
$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

$$(A \ominus kB) = ((\ldots((A \ominus B) \ominus B) \ominus \ldots) \ominus B)$$

$$K = \max\{k \mid (A \ominus kB) \neq \varnothing\}$$

Skeletons



a b
c d

FIGURE 9.23
(a) Set *A*.
(b) Various positions of maximum disks with centers on the skeleton of *A*.
(c) Another maximum disk on a different segment of the skeleton of *A*.
(d) Complete skeleton.

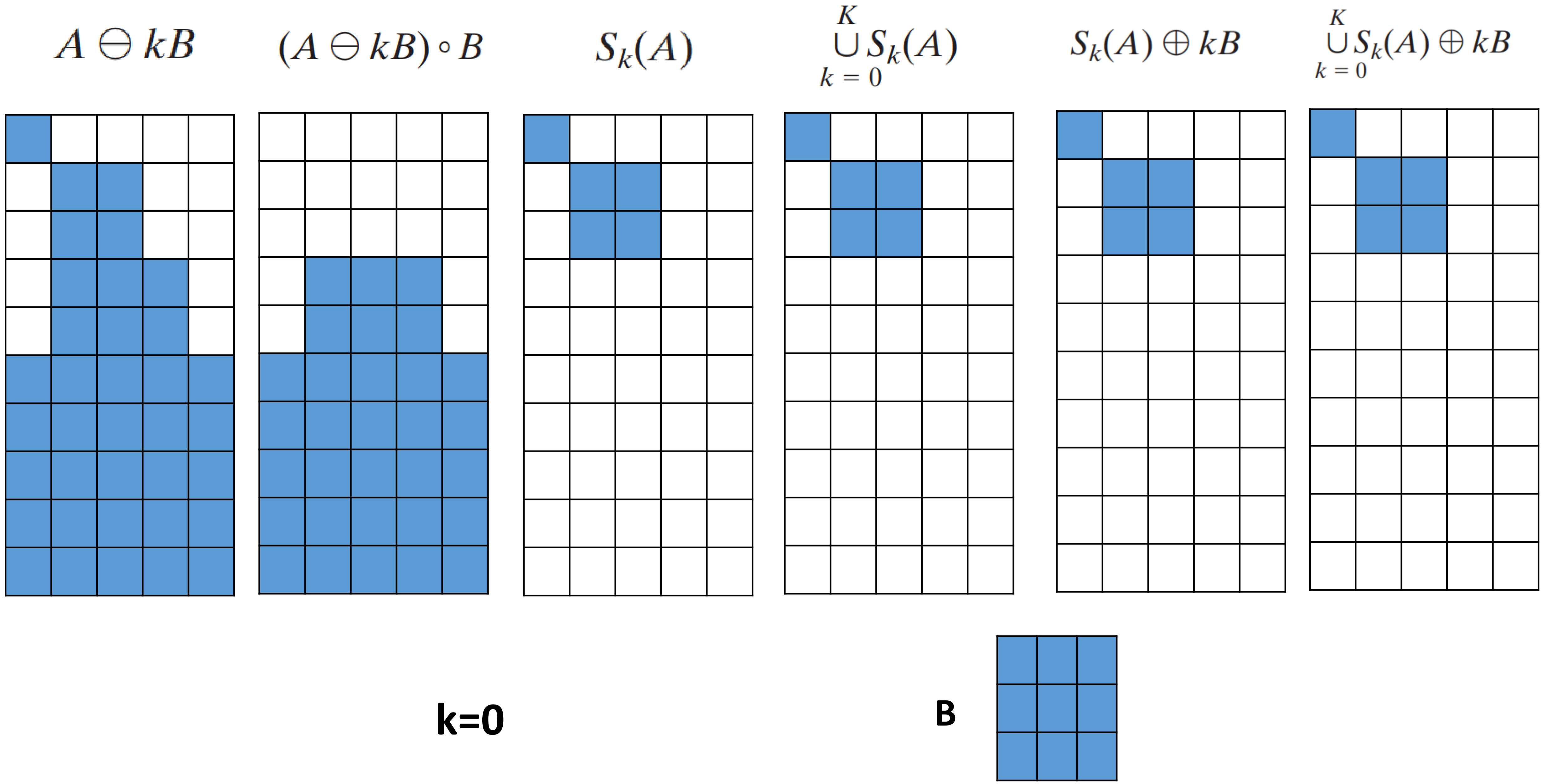
Skeletons

- Using skeleton we can retrieved original points set A

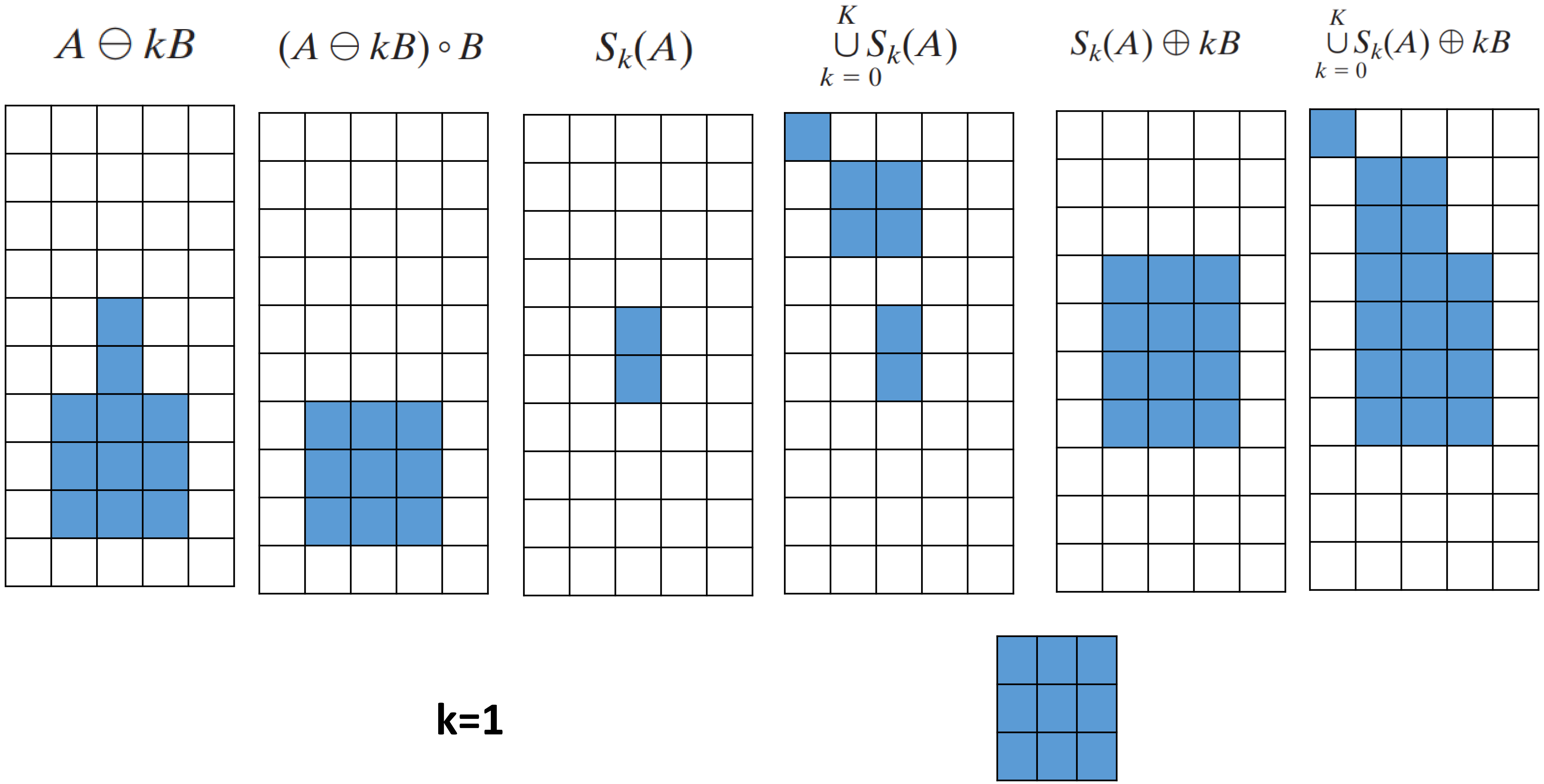
$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

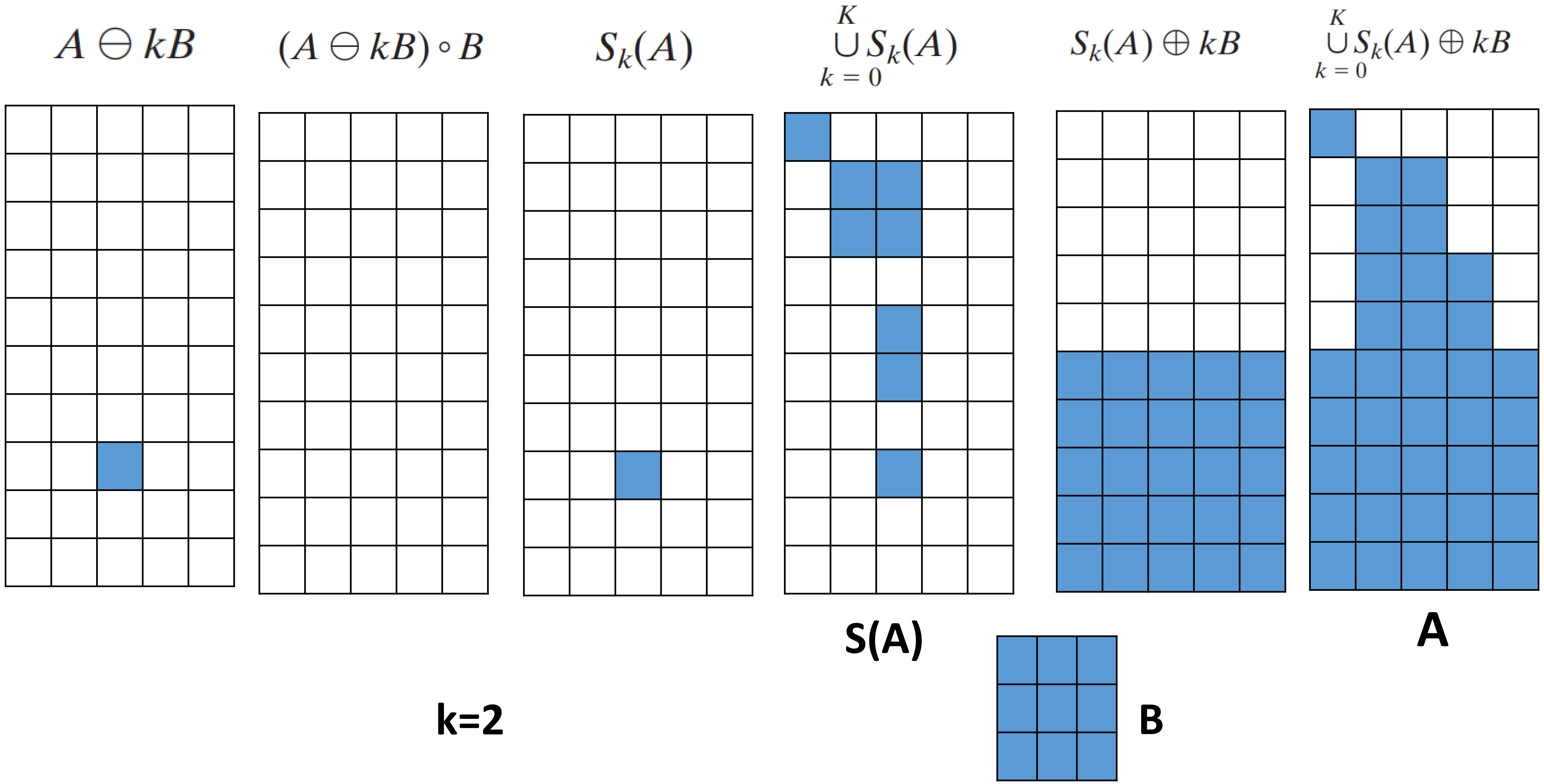
where $(S_k(A) \oplus kB)$ denotes k successive dilations of $S_k(A)$; that is,

$$(S_k(A) \oplus kB) = ((\ldots((S_k(A) \oplus B) \oplus B) \oplus \ldots) \oplus B)$$



$A \ominus kB$
$(A \ominus kB) \circ B$
$S_k(A)$
$\bigcup_{k=0}^{K} S_k(A)$
$S_k(A) \oplus kB$
$\bigcup_{k=0}^{K} S_k(A) \oplus kB$
k=0
B


$A \ominus kB$
$(A \ominus kB) \circ B$
$S_k(A)$
$\bigcup_{k=0}^{K} S_k(A)$
$S_k(A) \oplus kB$
$\bigcup_{k=0}^{K} S_k(A) \oplus kB$
k=1



$A \ominus kB$
$(A \ominus kB) \circ B$
$S_k(A)$
$\bigcup_{k=0}^{K} S_k(A)$
$S_k(A) \oplus kB$
$\bigcup_{k=0}^{K} S_k(A) \oplus kB$
S(A)
A
k=2
B

Convex Hull

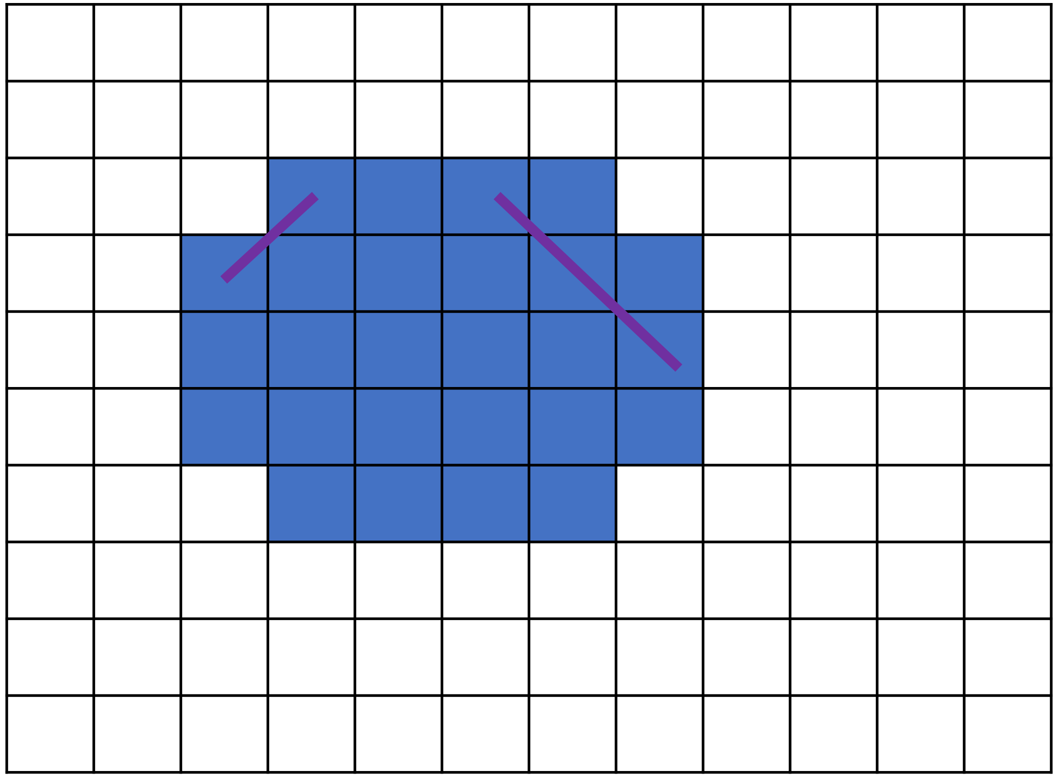
- Set A is said to be ***convex*** if the straight line segment joining any two points in A lies entirely within A

- The convex hull H of an arbitrary set S is the smallest convex set containing S.

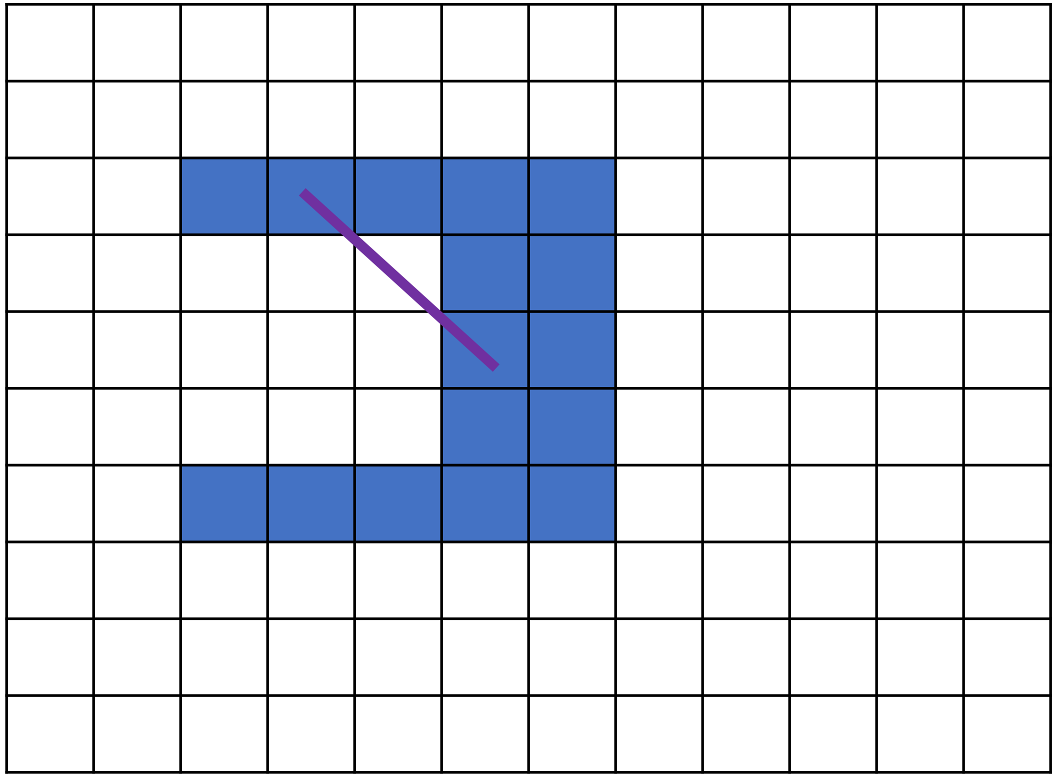
- The set difference $H - S$ is called the convex deficiency of S

Convex Hull

- Convex set



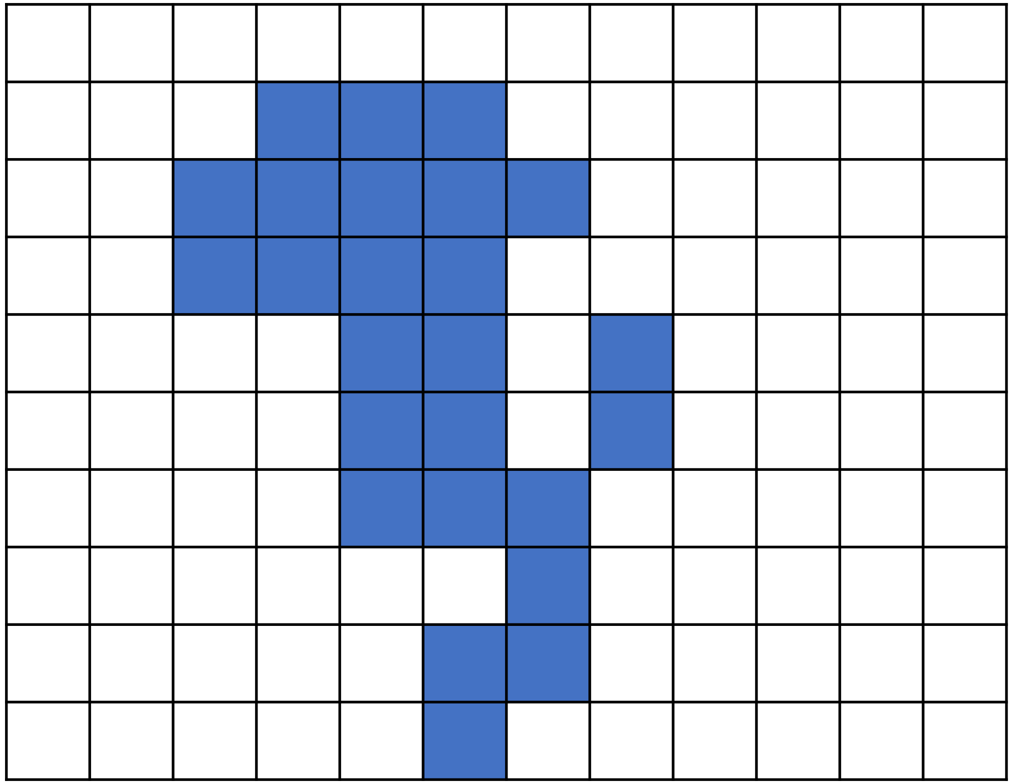
Convex set S_1



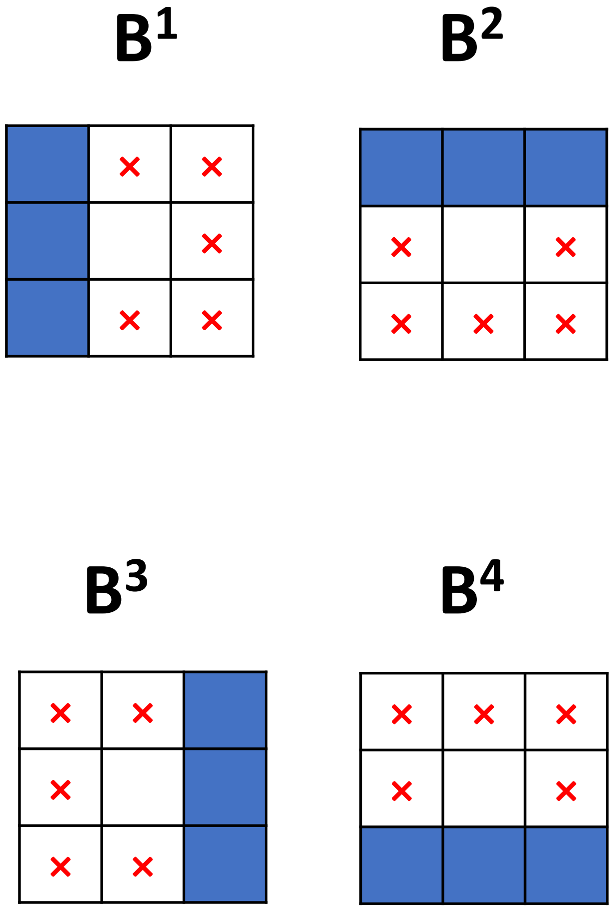
Not Convex set S_2

- Perform four structure element B^i
- $X_k^i = (X_{k-1}^i \circledast B^i) \cup A$ where i=1,2,3,4 and k = 1,2,3,4,...
- $X_0^i = A$
- Point of convergence $X_k^i = X_{k-1}^i$
- Let $D^i = X_k^i$
- The convex hull of A is $C(A) = \bigcup_{i=1}^{4} D^i$

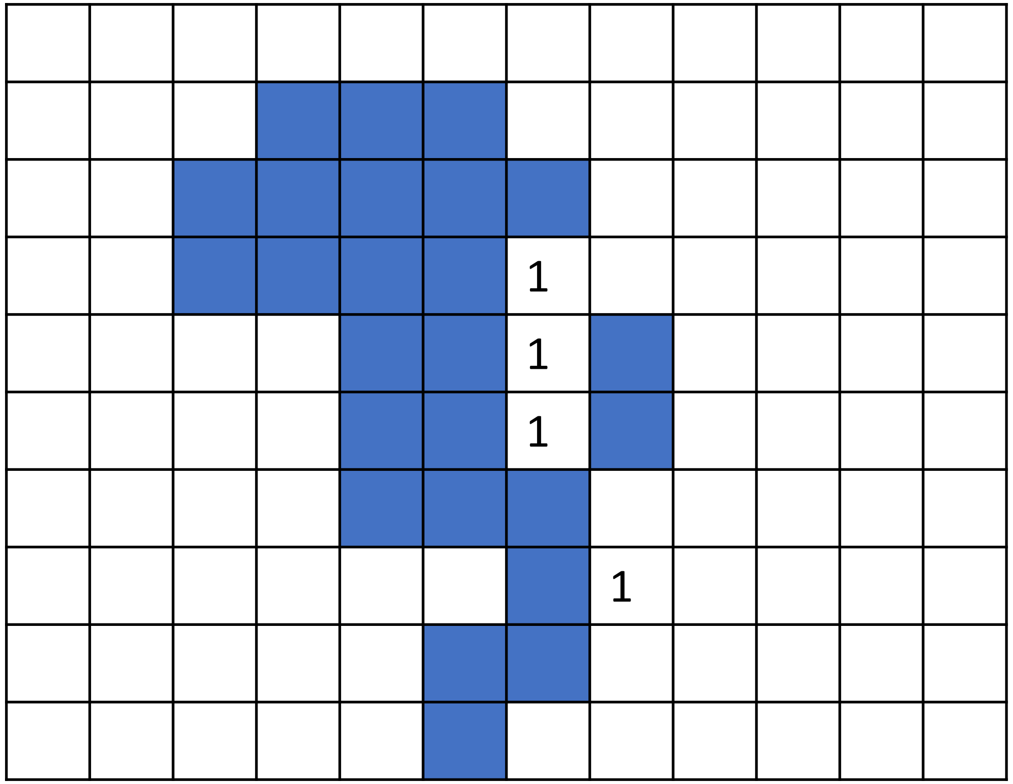
Convex Hull



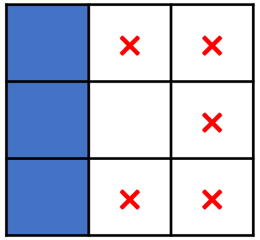
Set A



Convex Hull



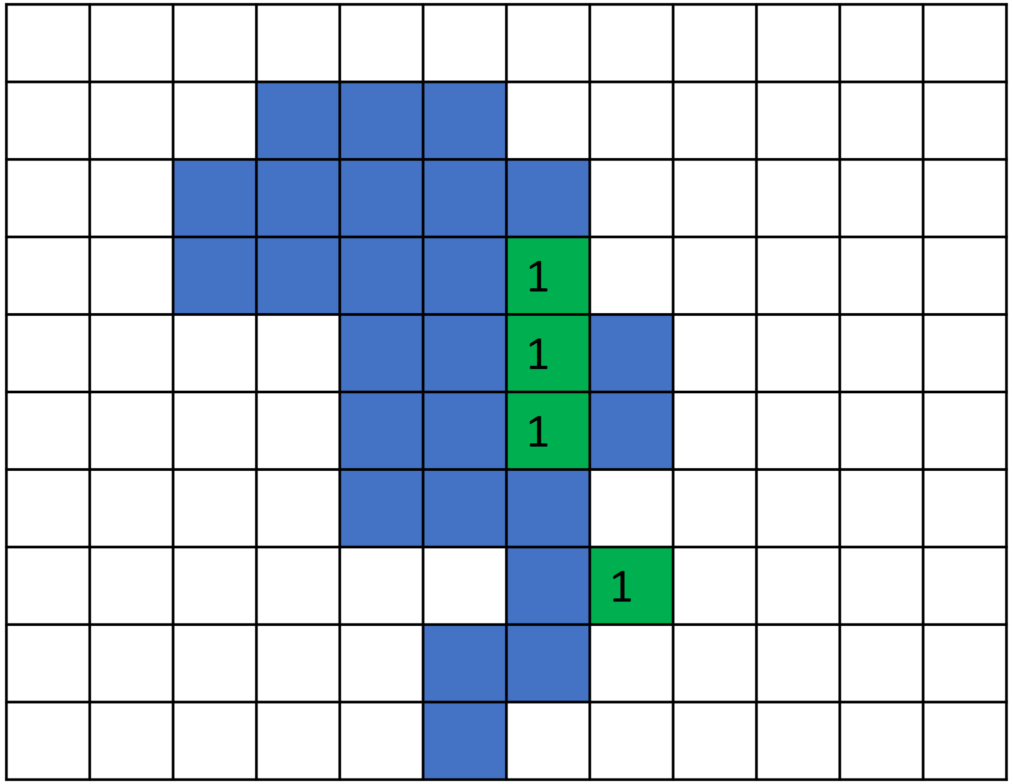
B^1



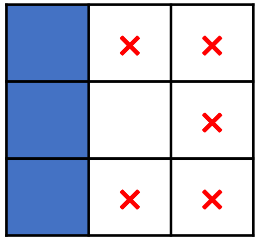
Iteration 1

X_1^1

Convex Hull



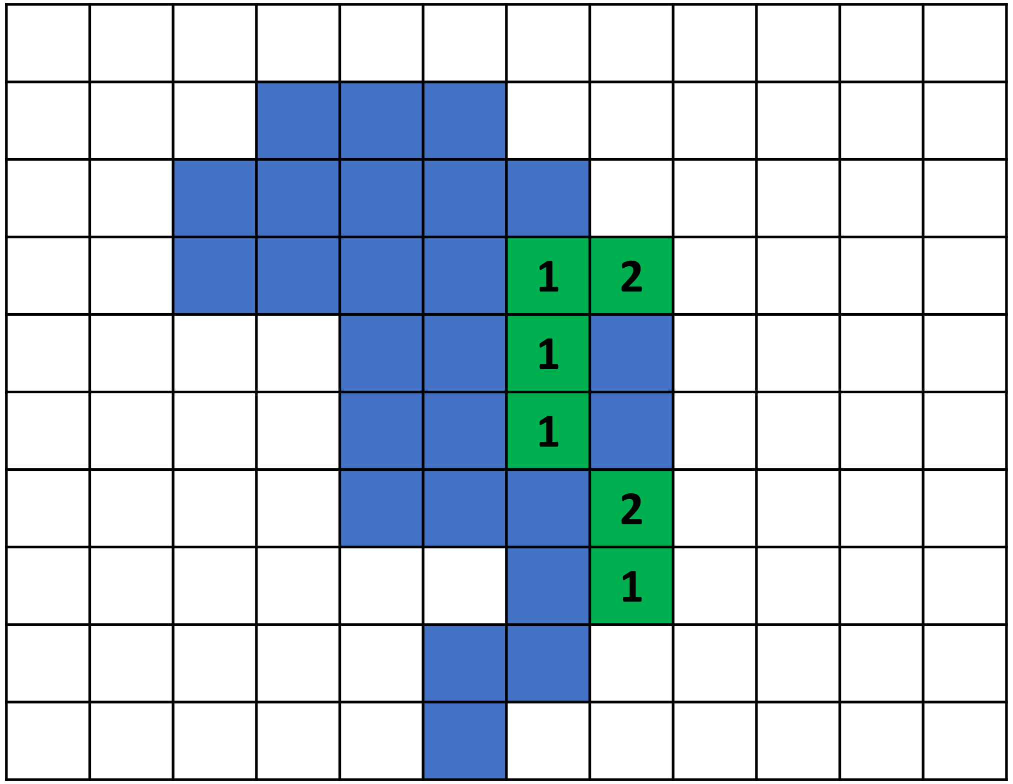
B^1



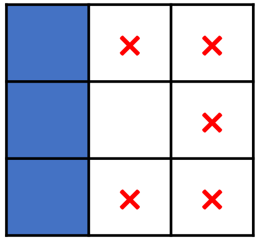
Iteration 1

X_1^1

Convex Hull



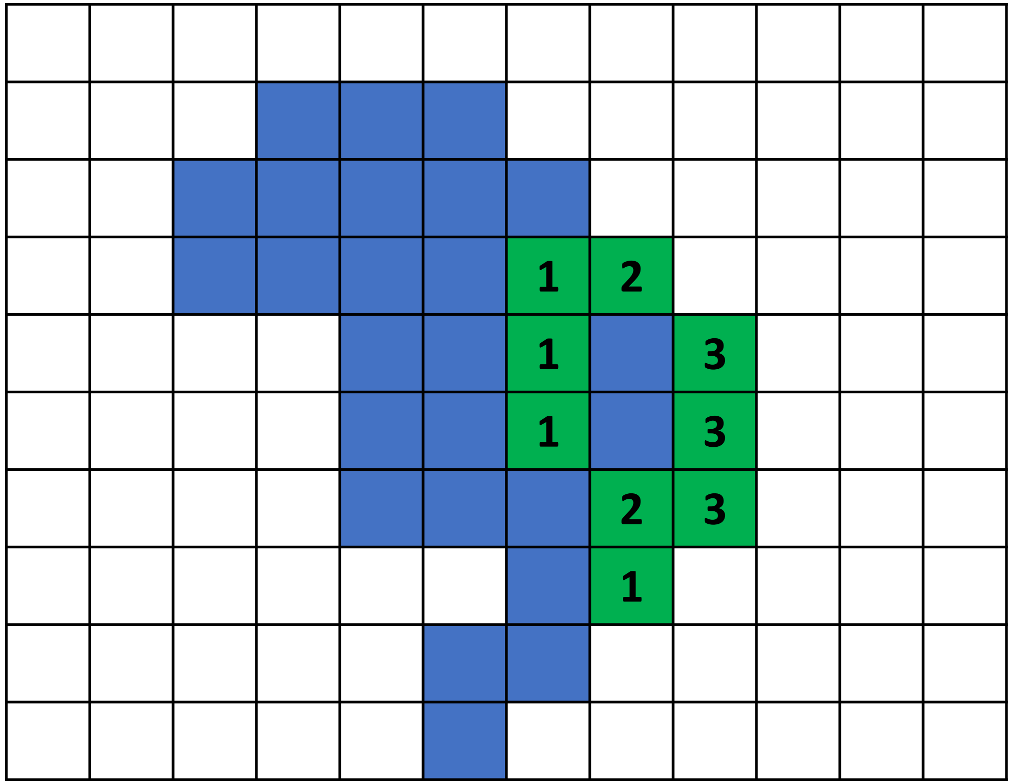
B^1



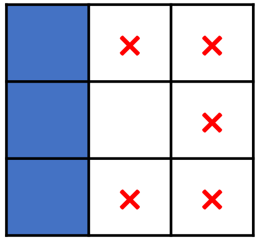
Iteration 2

X_2^1

Convex Hull



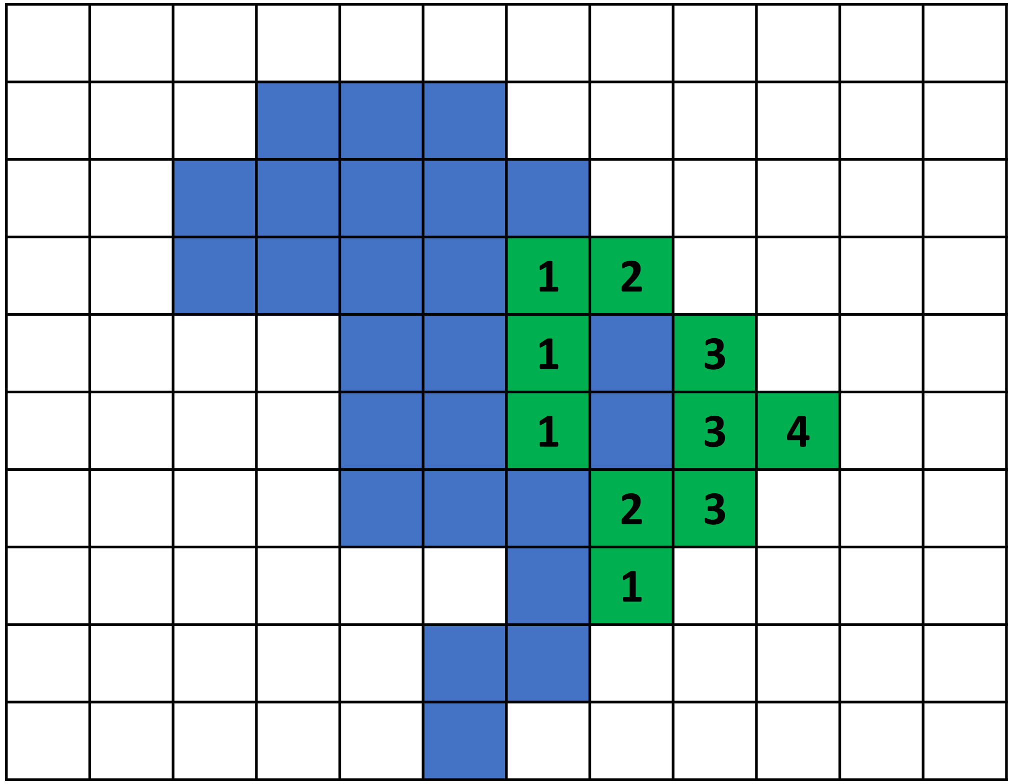
B^1



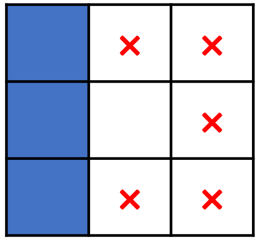
Iteration 3

X_3^1

Convex Hull



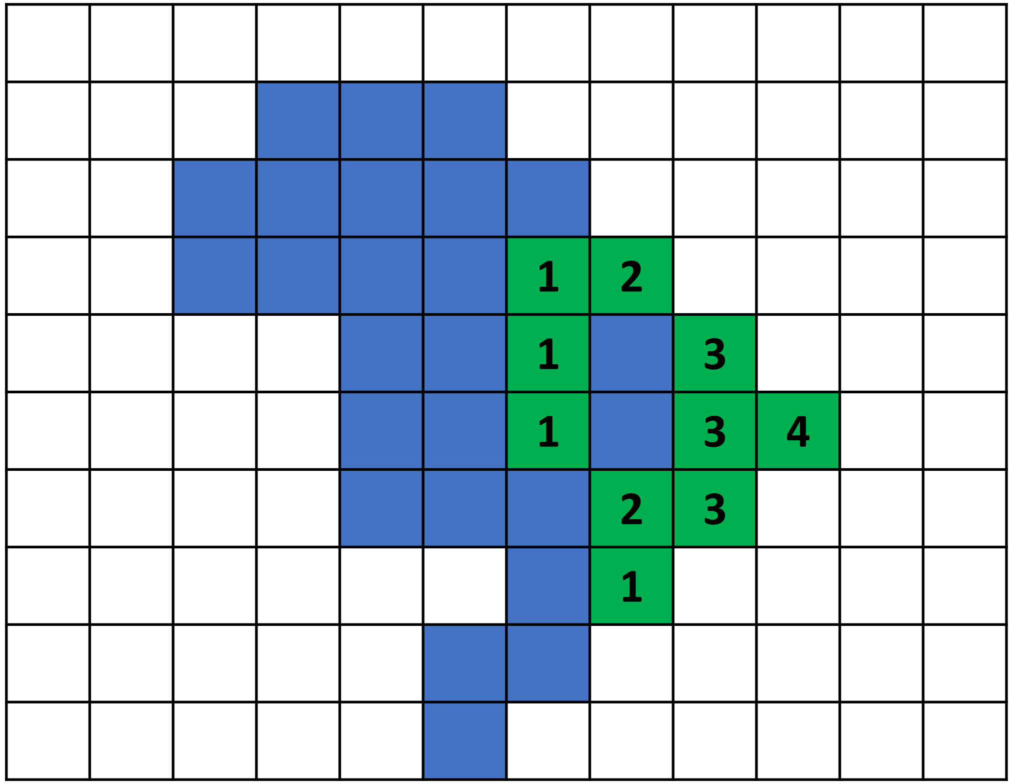
B^1



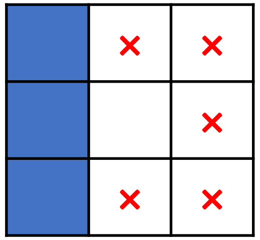
Iteration 4

X_4^1

Convex Hull



B^1

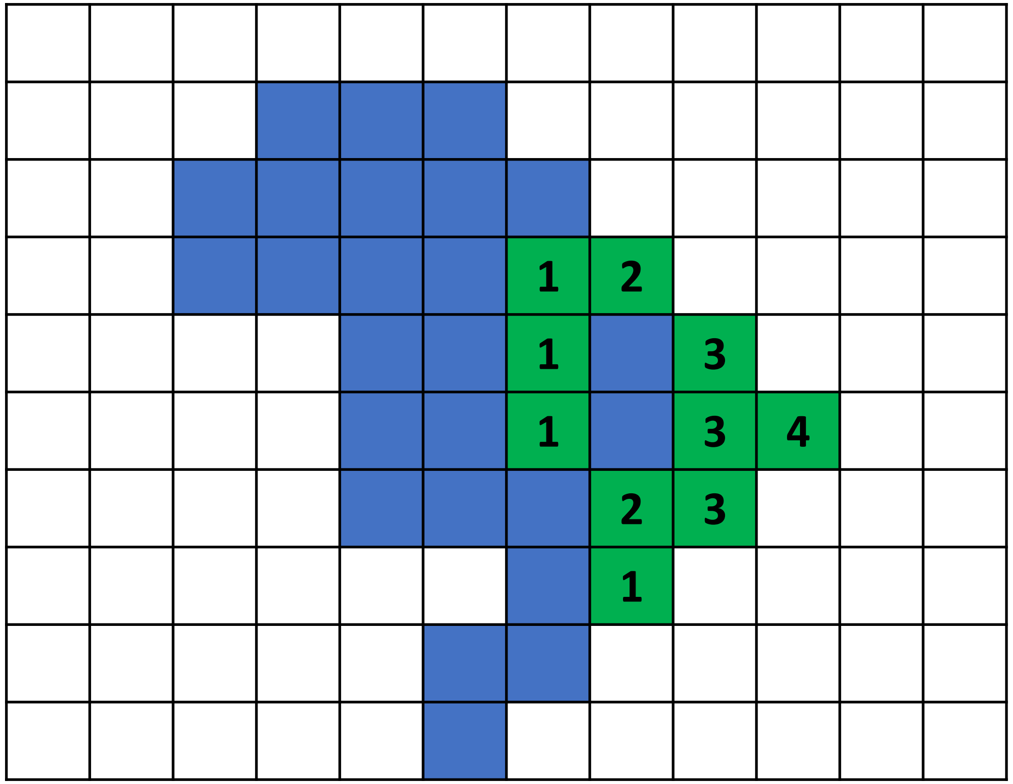


Iteration 5

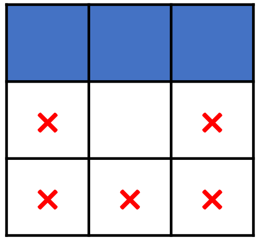
$X_5^1 = X_4^1$

Process terminate

Convex Hull



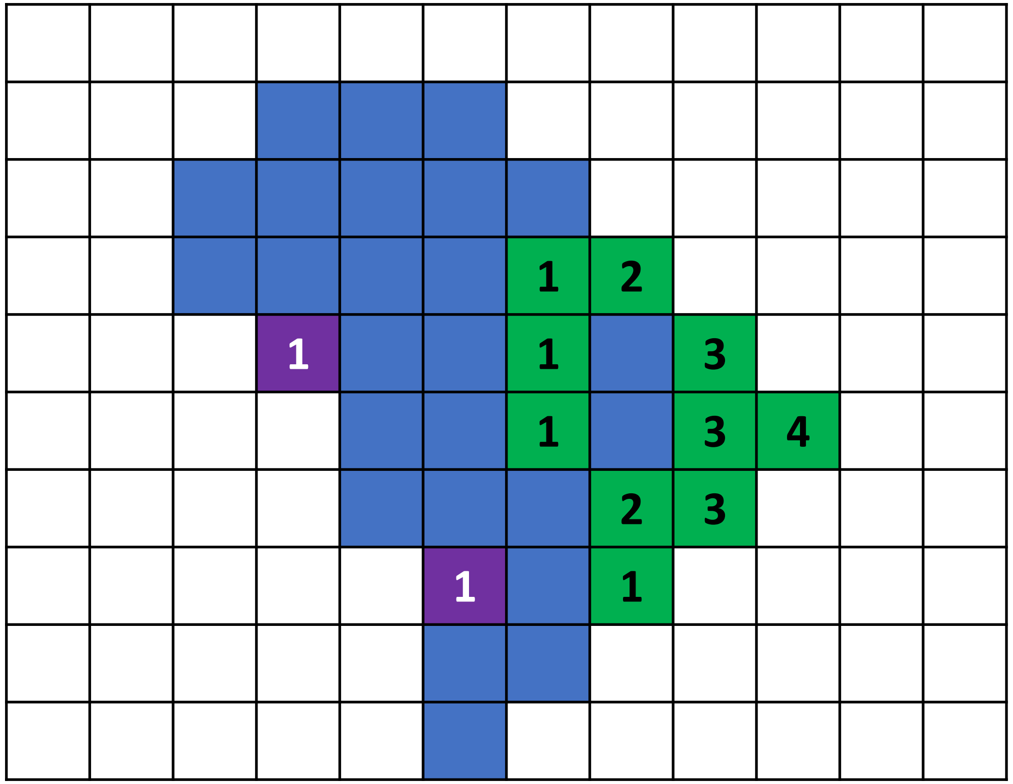
B^2



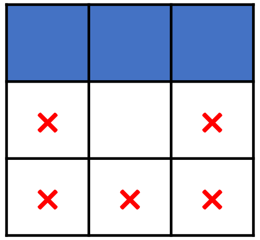
Iteration 1

X_1^2

Convex Hull



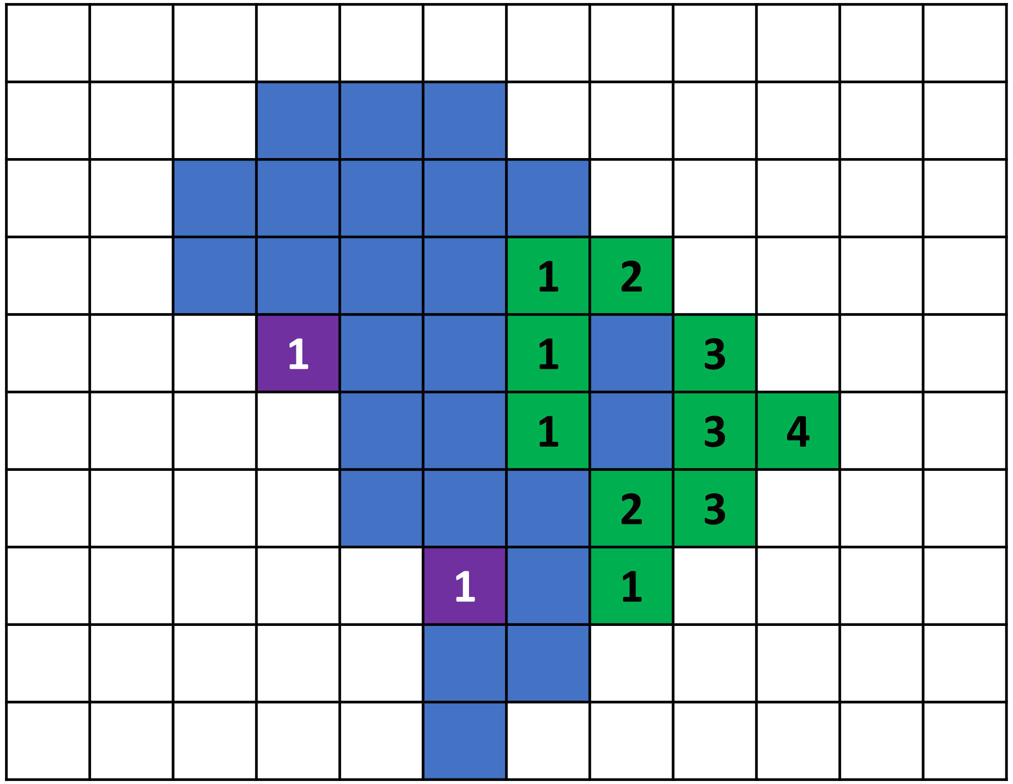
B^2



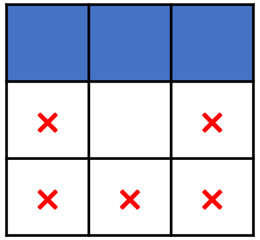
Iteration 1

X_1^2

Convex Hull



B^2

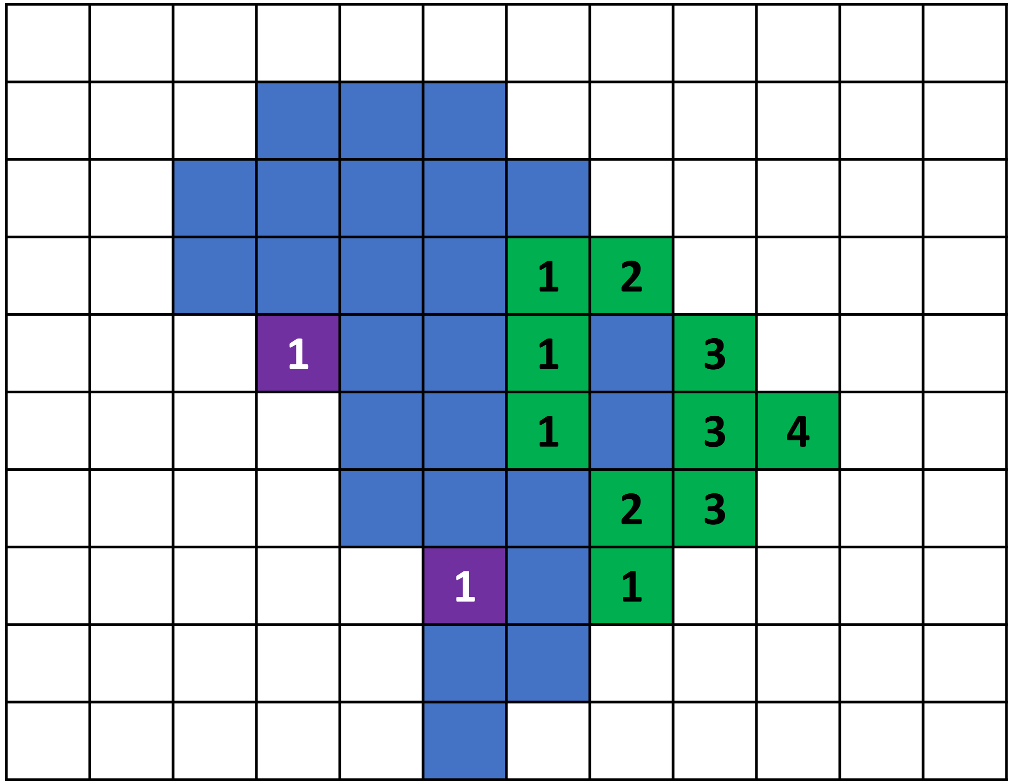


Iteration 2

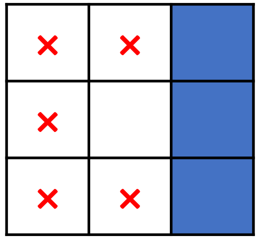
$X_2^2 = X_1^2$

Process terminate

Convex Hull



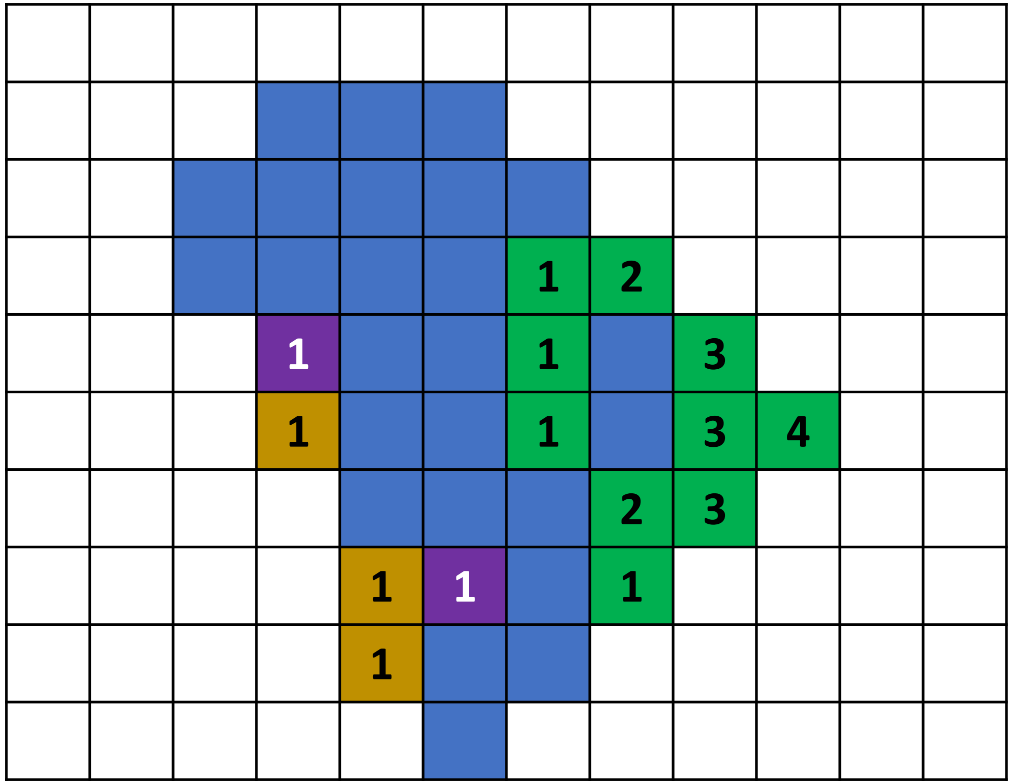
B^3



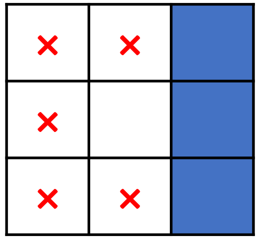
Iteration 1

X_1^3

Convex Hull



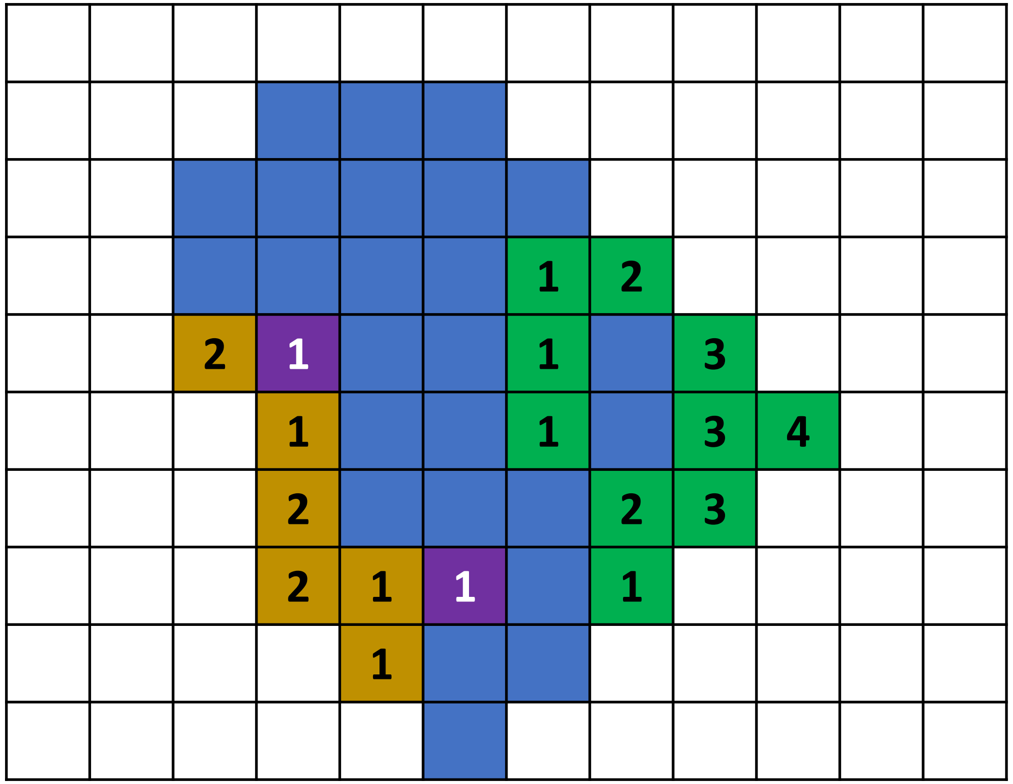
B^3

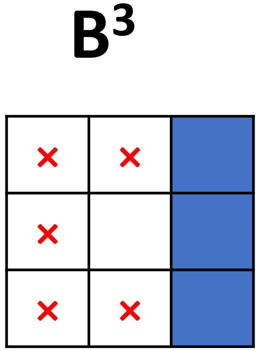


Iteration 1

X_1^3

Convex Hull

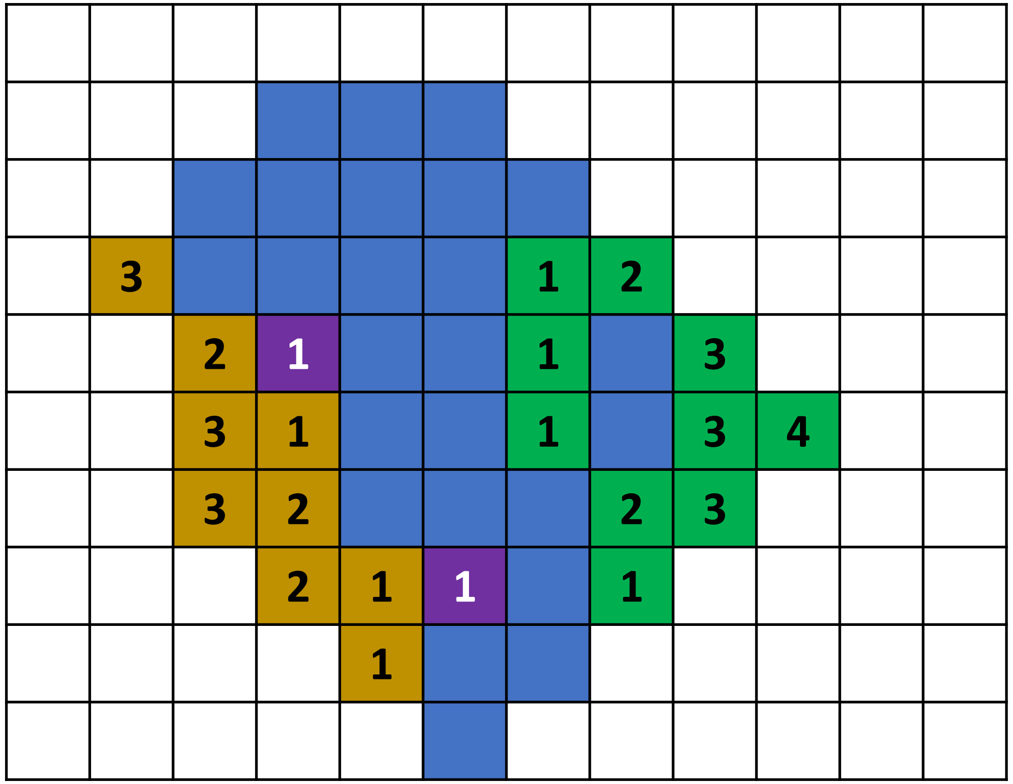




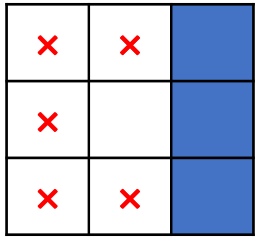
Iteration 2

X_2^3

Convex Hull



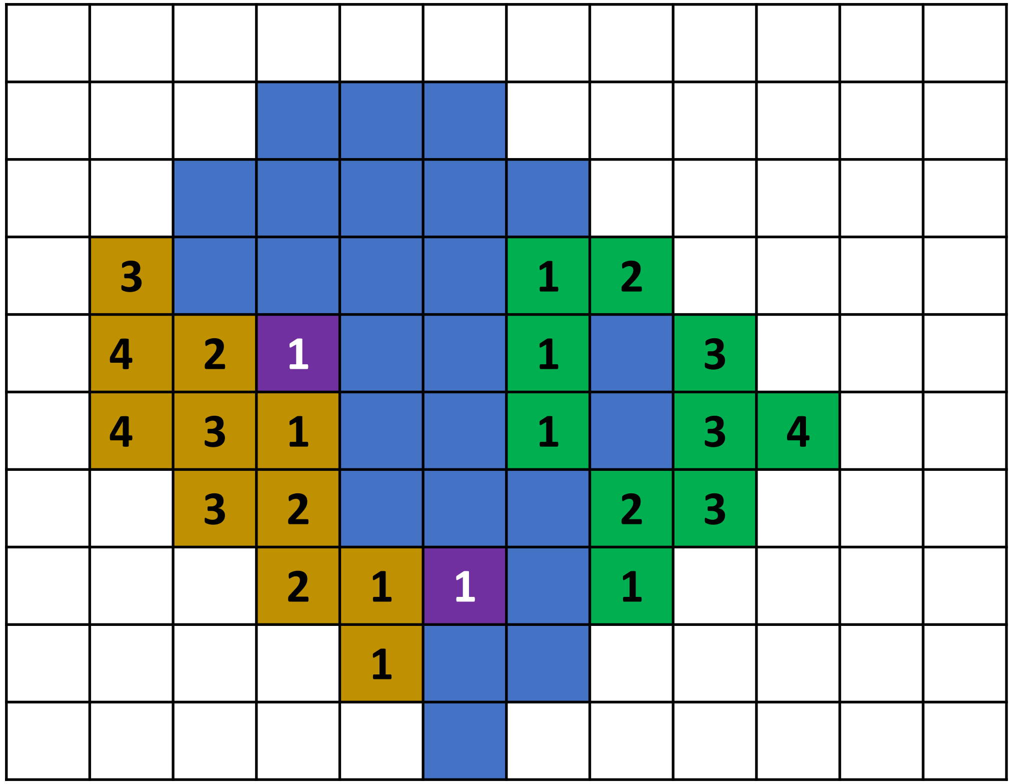
B^3



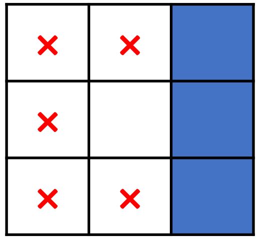
Iteration 3

X_3^3

Convex Hull



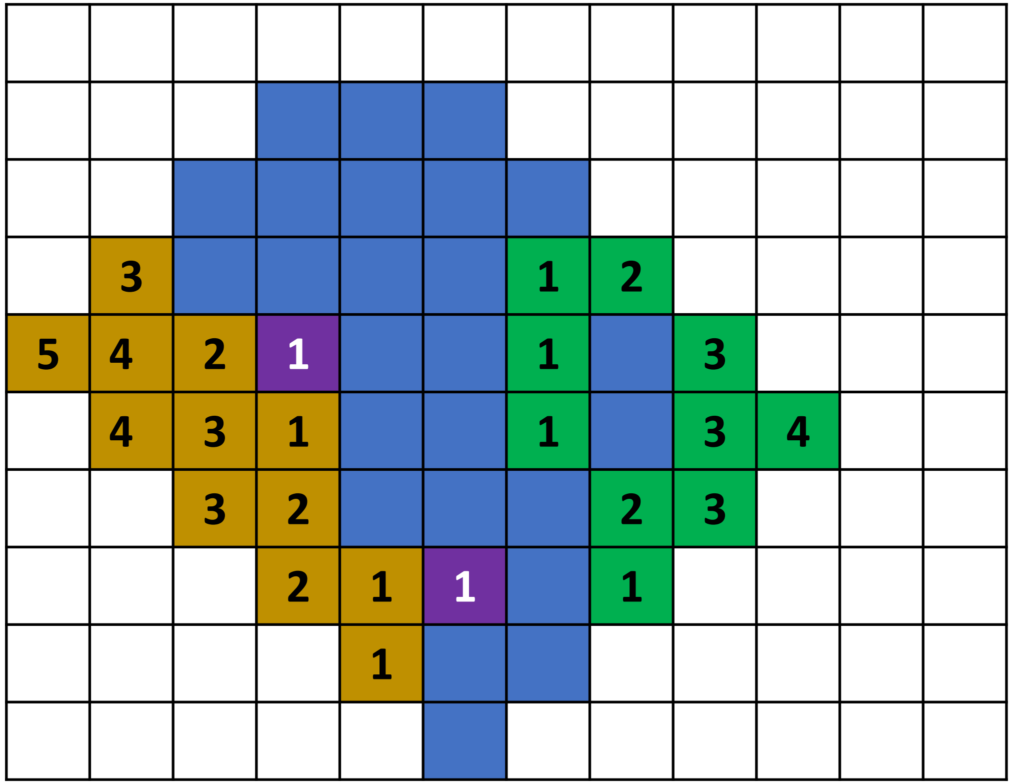
B^3



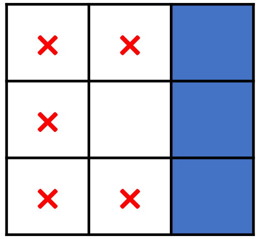
Iteration 4

X_4^3

Convex Hull



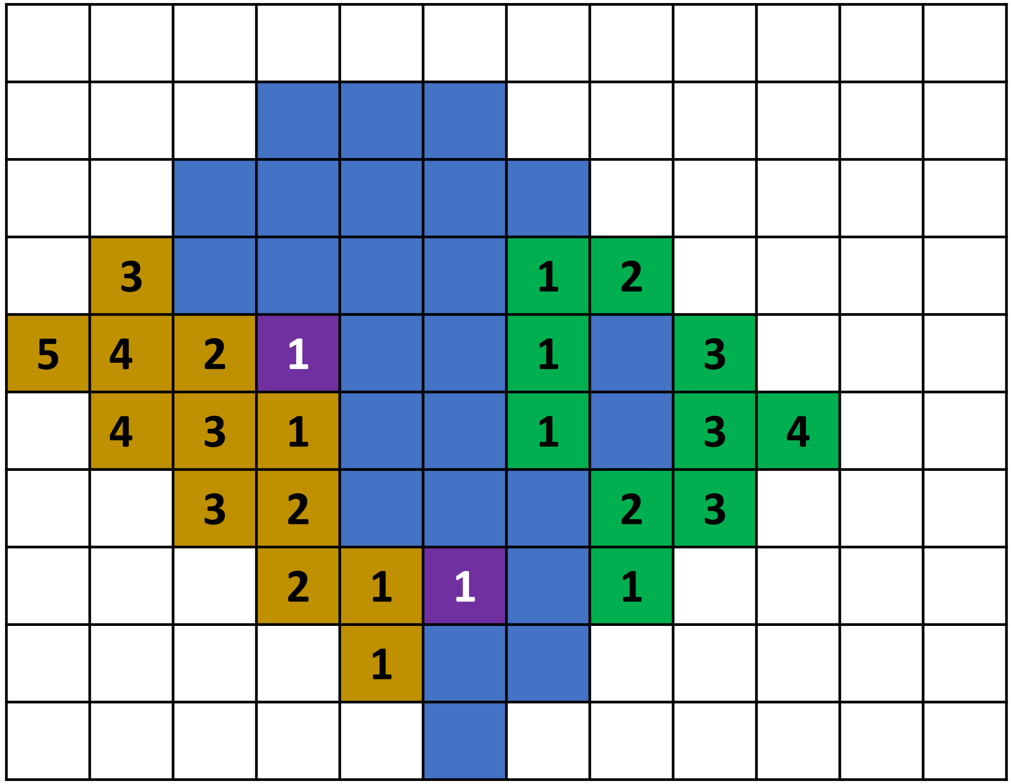
B^3



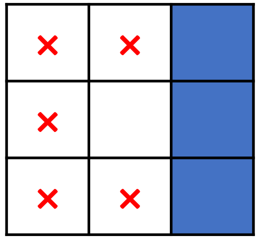
Iteration 5

X_5^3

Convex Hull



B^3

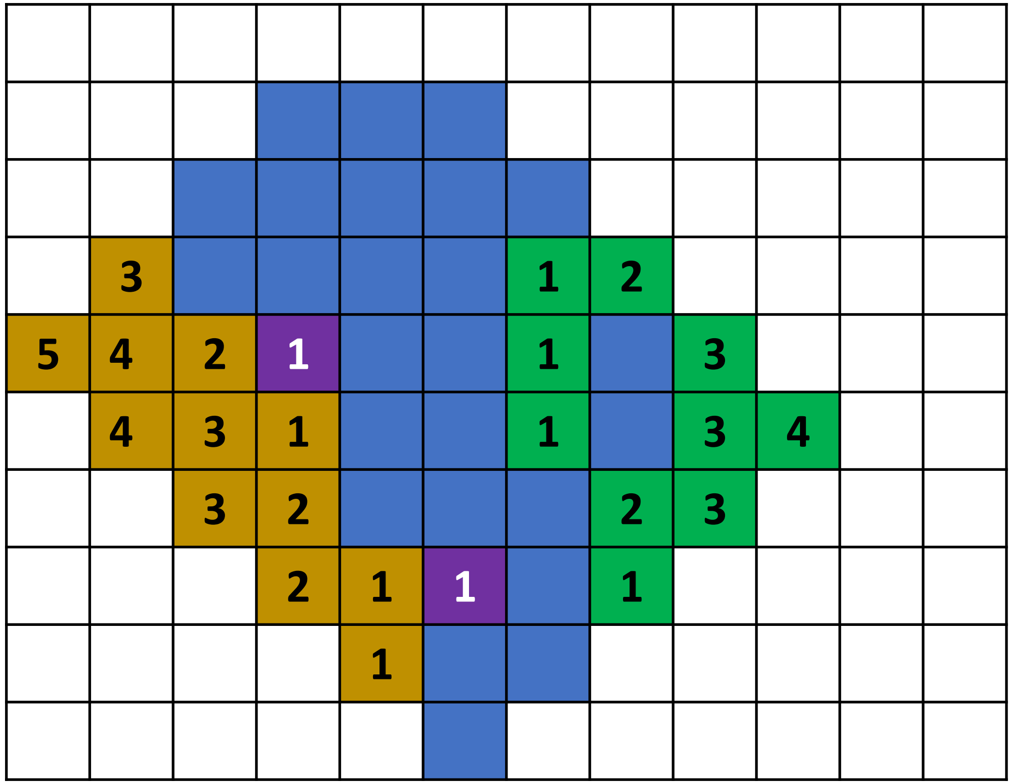


Iteration 6

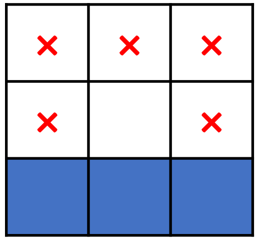
$X_6^3 = X_5^3$

Process terminate

Convex Hull



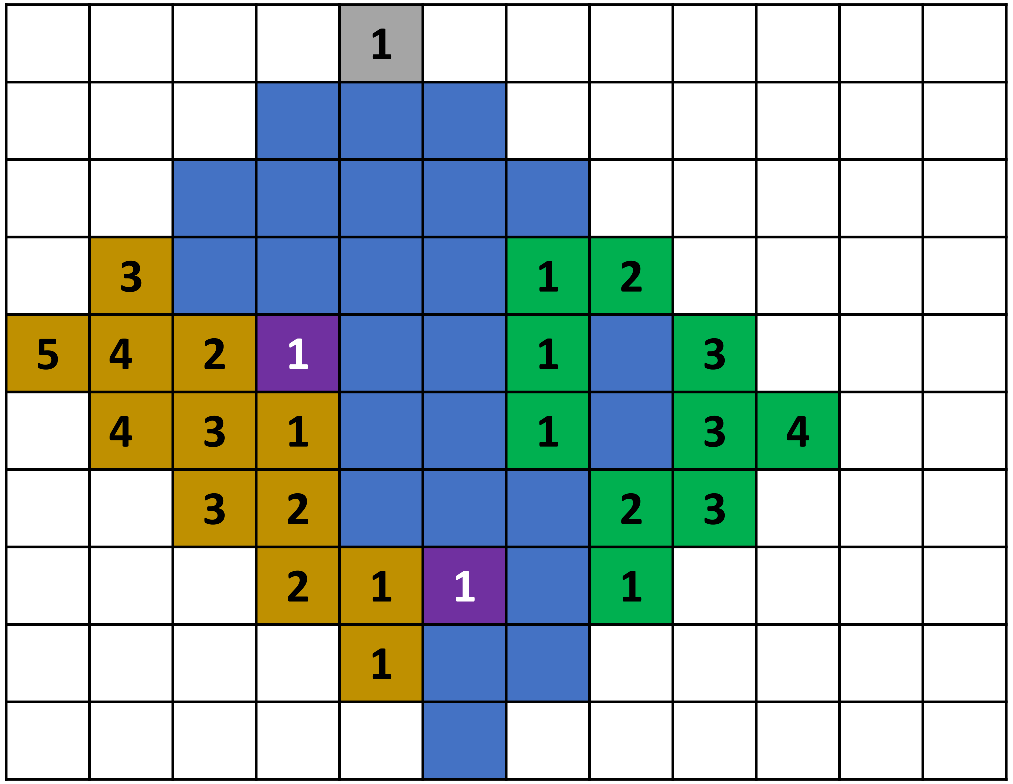
B^4



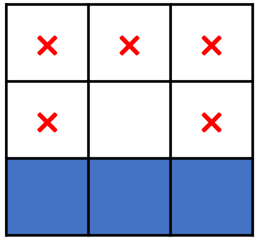
Iteration 1

X_1^4

Convex Hull



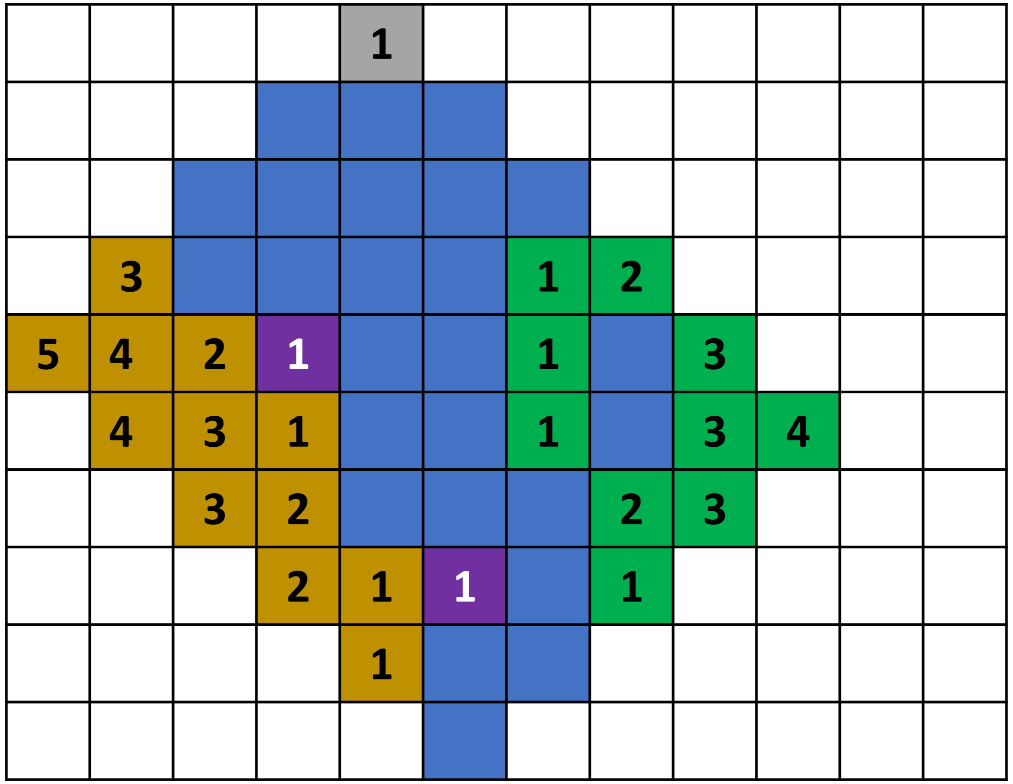
B^4



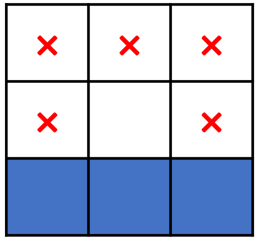
Iteration 1

X_1^4

Convex Hull



B^4



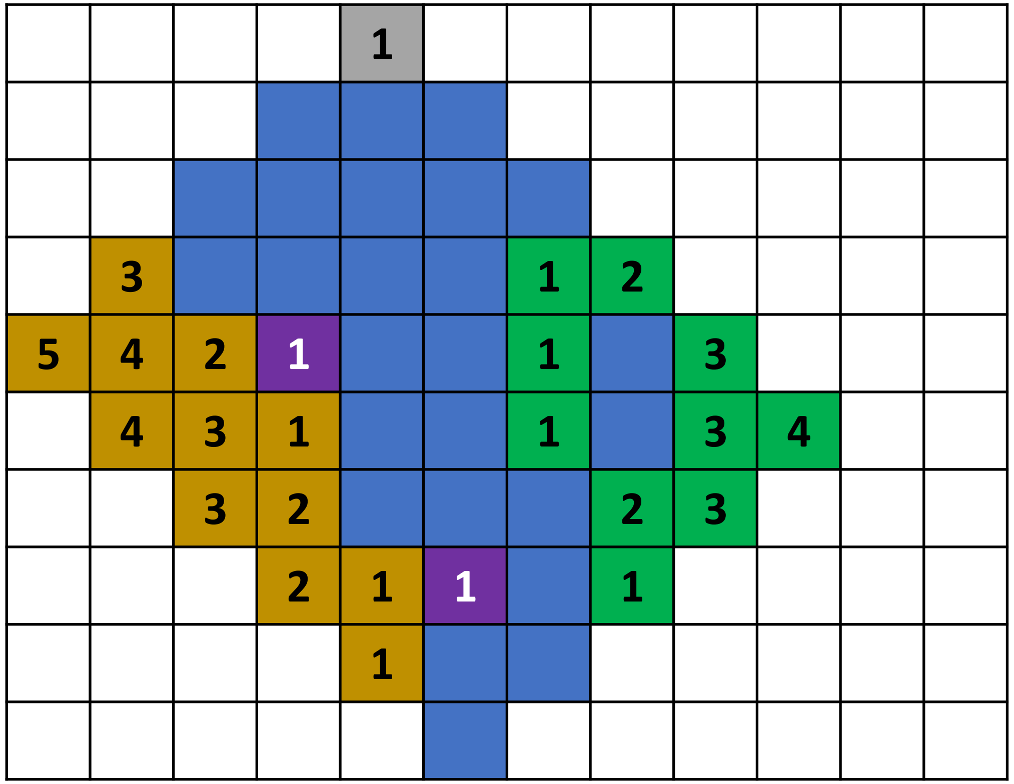
Iteration 2

$X_2^4 = X_1^4$

Process terminate

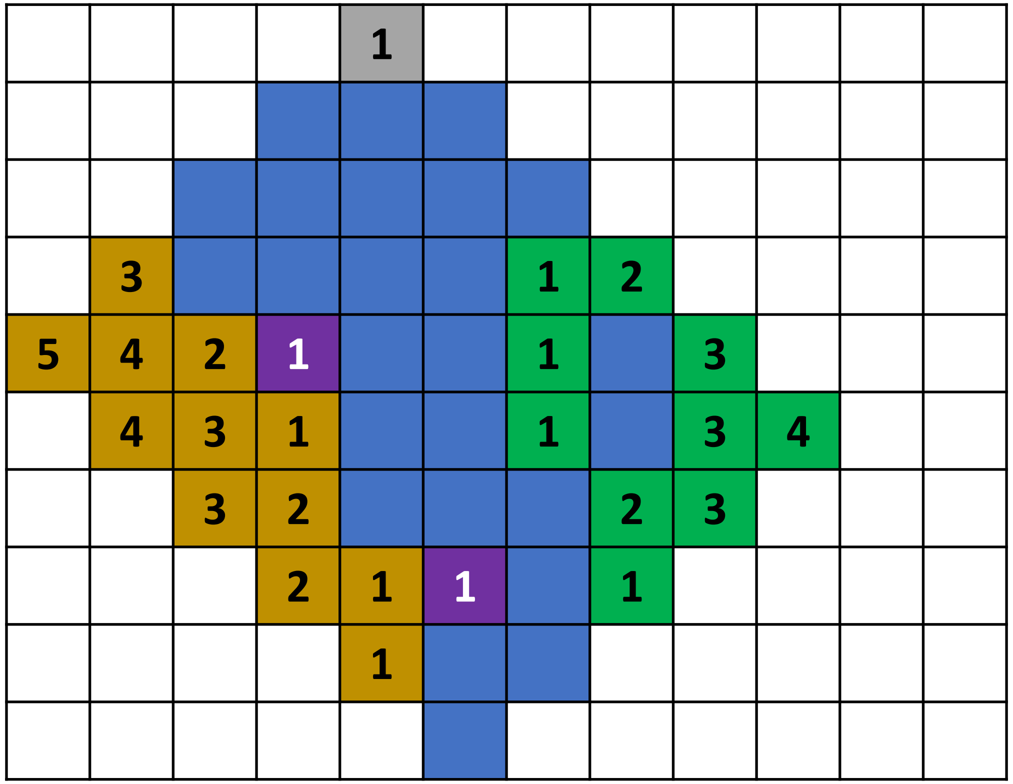
Convex Hull

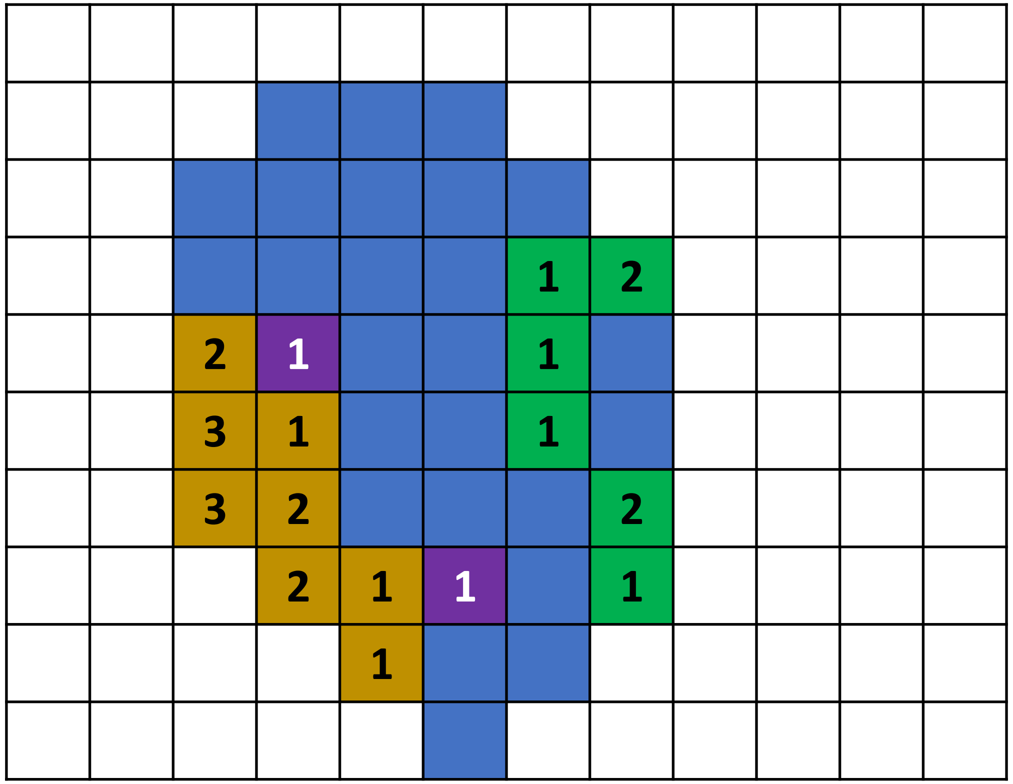
- So the convex hull is



Convex Hull drawbacks

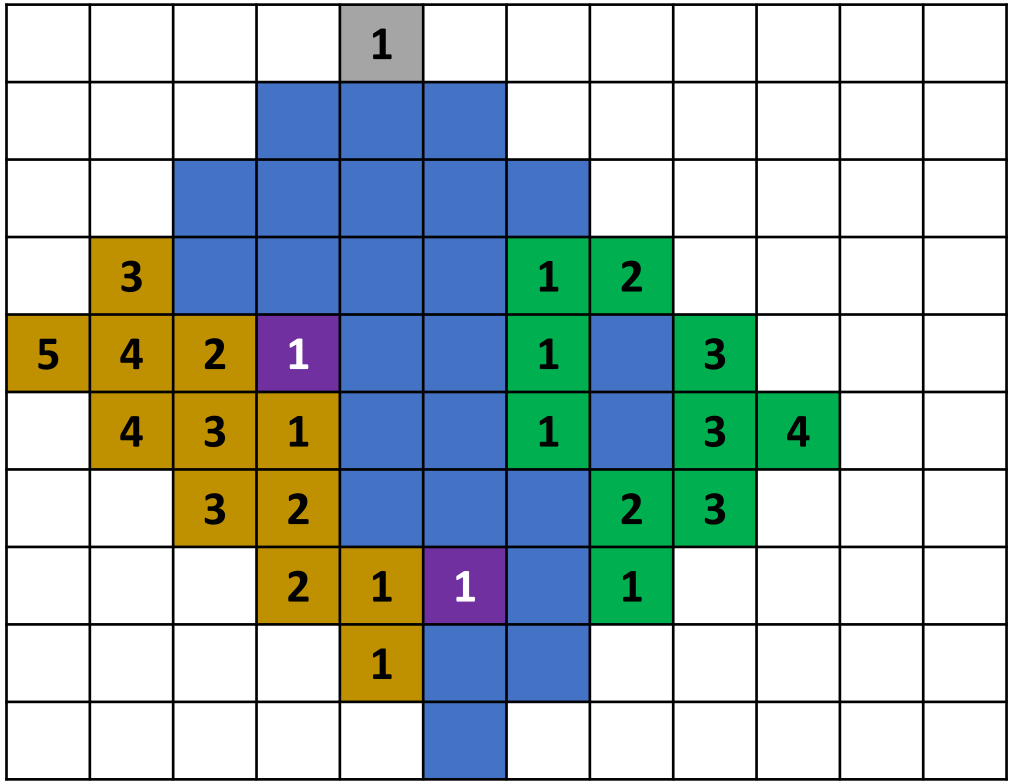
- Convex hull is the minimal of convex set

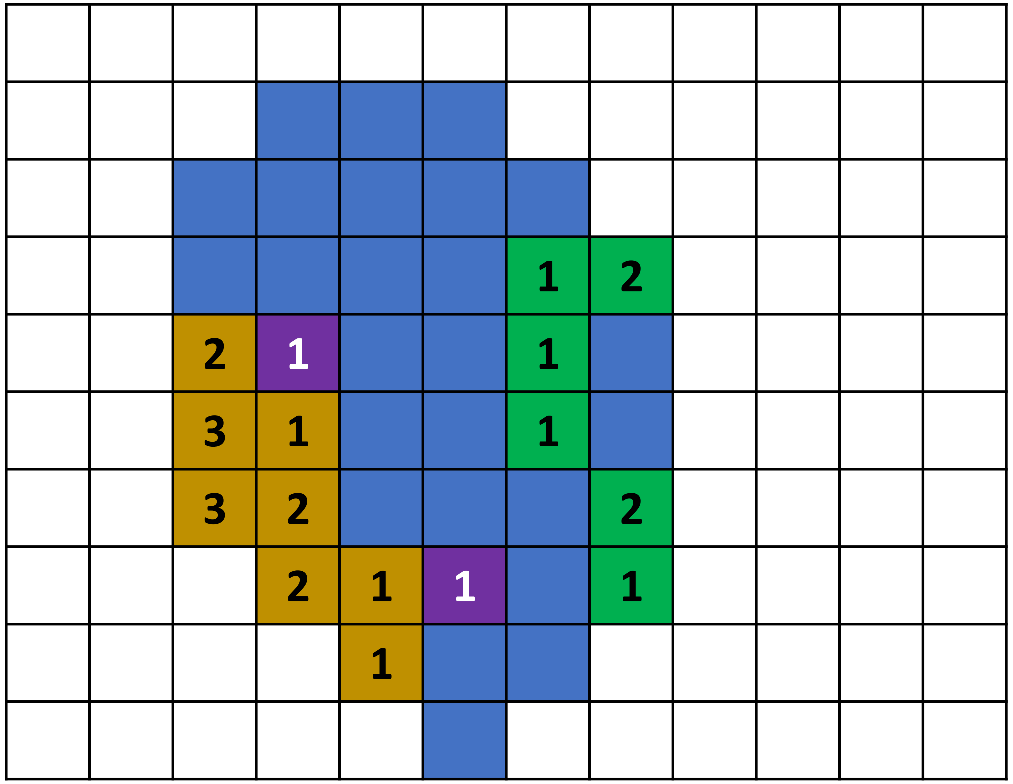




Convex Hull drawbacks solution

- Limit horizontal, vertical dimensions





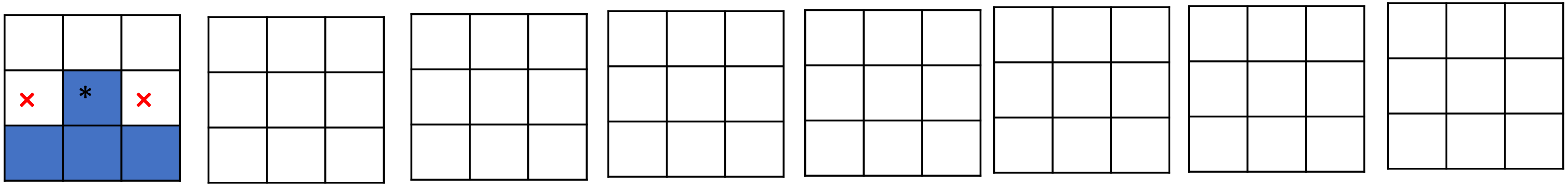
Thinning

- The thinning of set A by a structuring element B
  - $A \otimes B = A - (A \circledast B)$

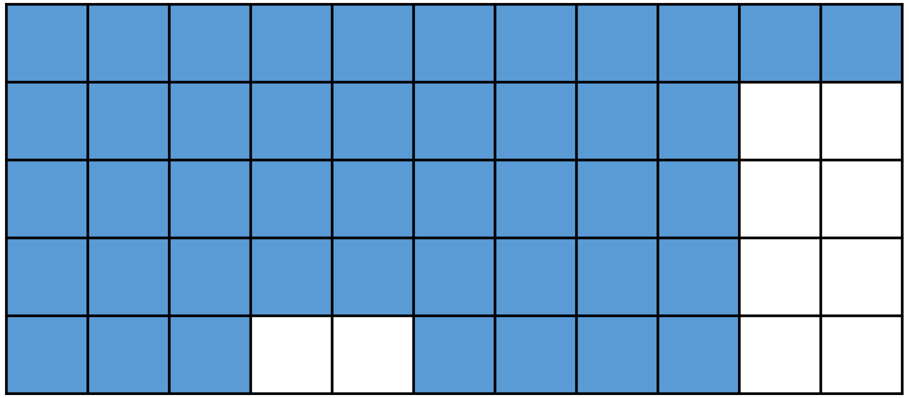
$$= A \cap (A \circledast B)^c$$

- A sequence of structuring element $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$
- Every B^i is the rotated version of B^{i-1}
- Thinning is defined by $A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$

Thinning

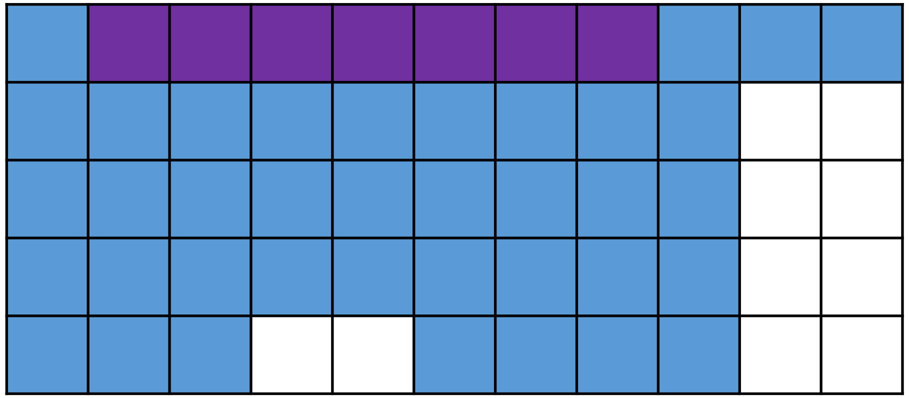


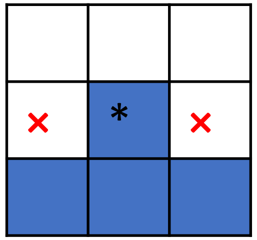
B^1 B^2 B^3 B^4 B^5 B^6 B^7 B^8



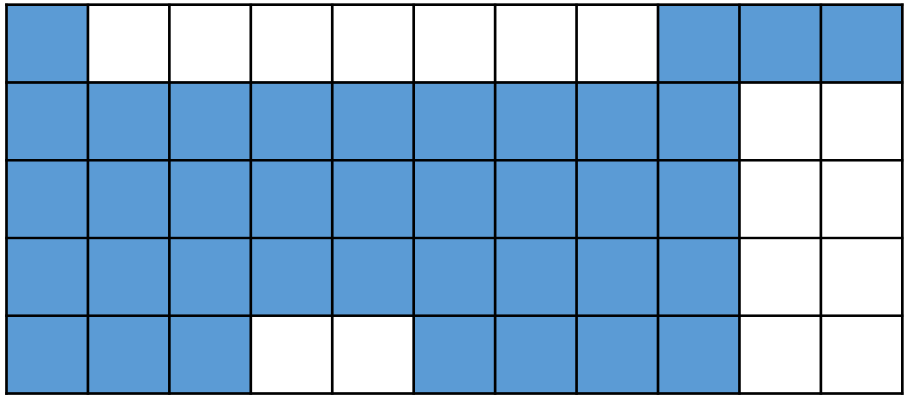
Set A

Thinning

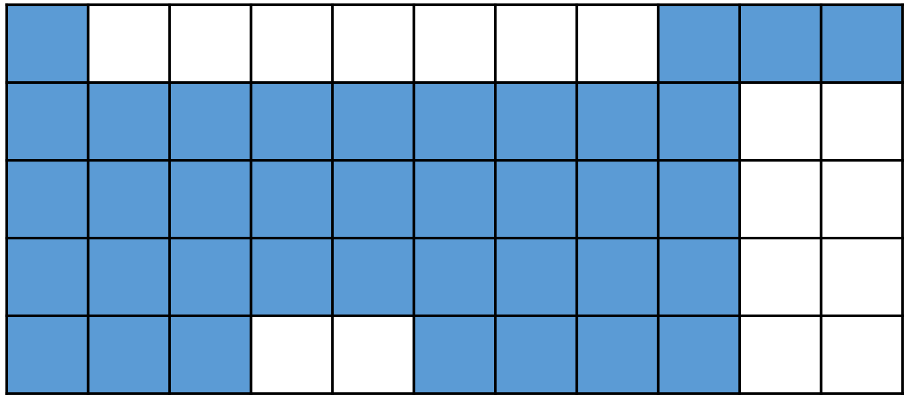


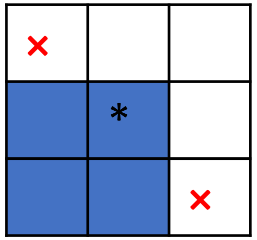


B^1

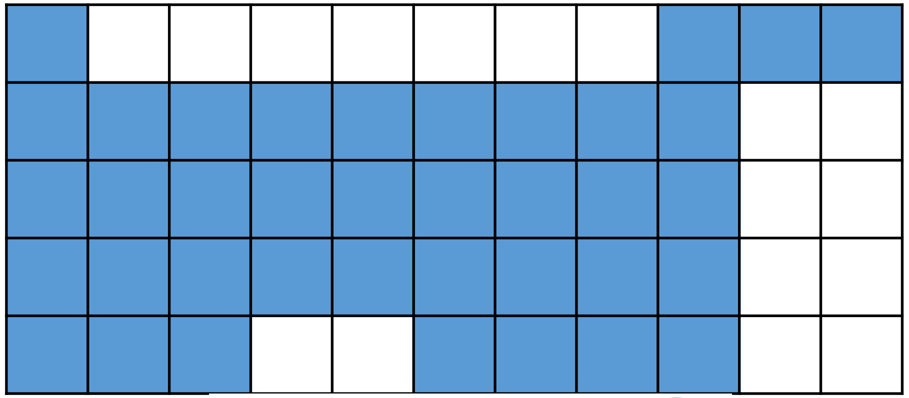


$$A_1 = A \otimes B^1$$



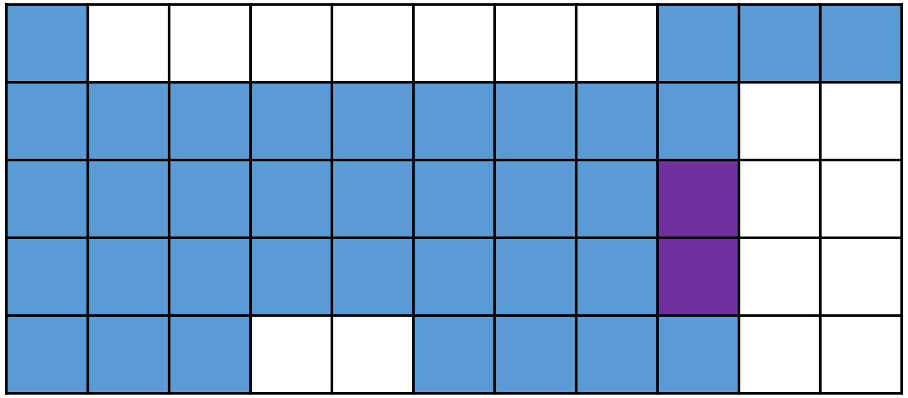


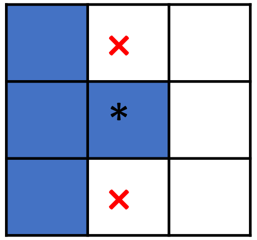
B^2



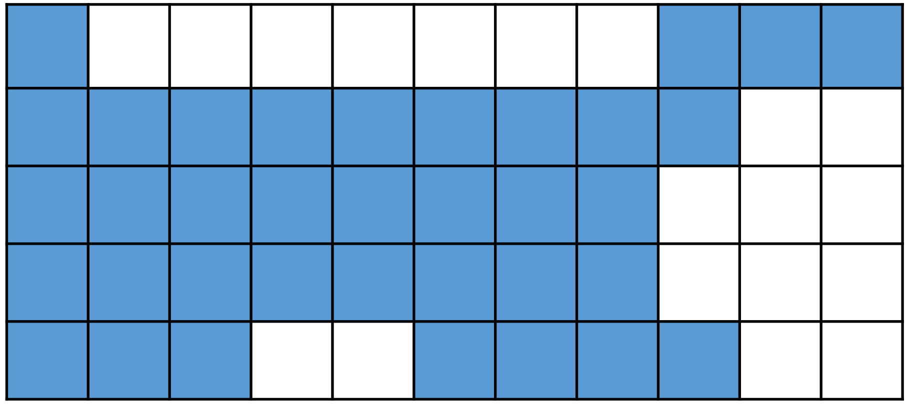
$$A_2 = A_1 \otimes B^2$$

Thinning

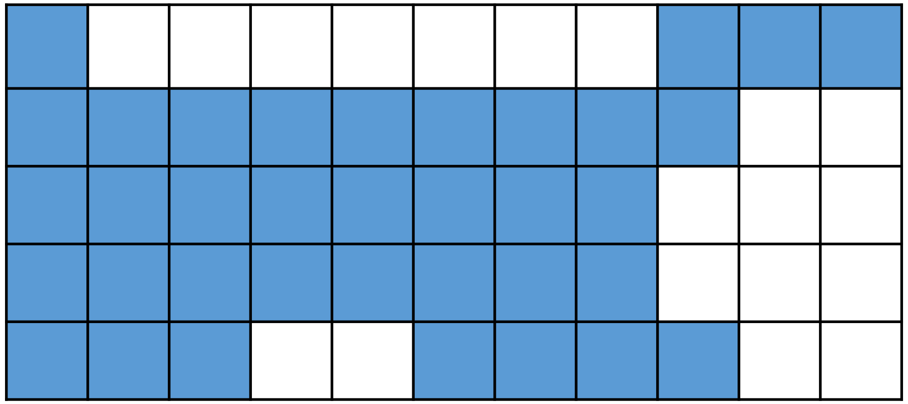


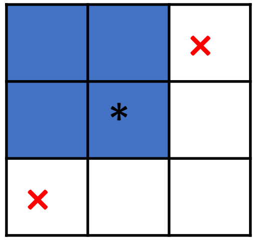


B^3

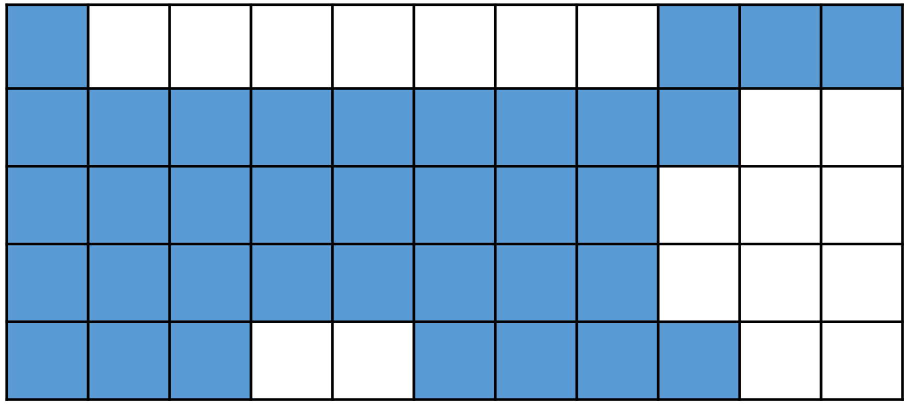


$$A_3 = A_2 \otimes B^3$$



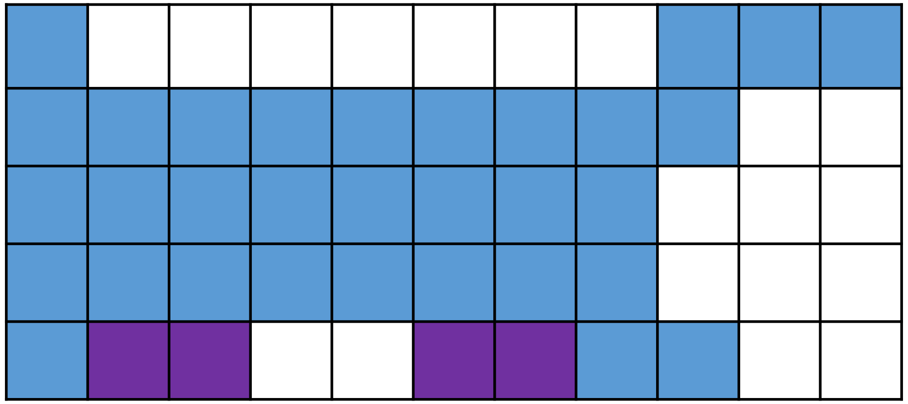


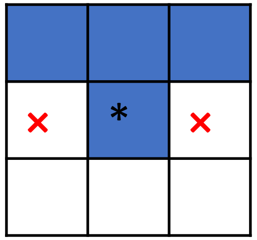
B^4



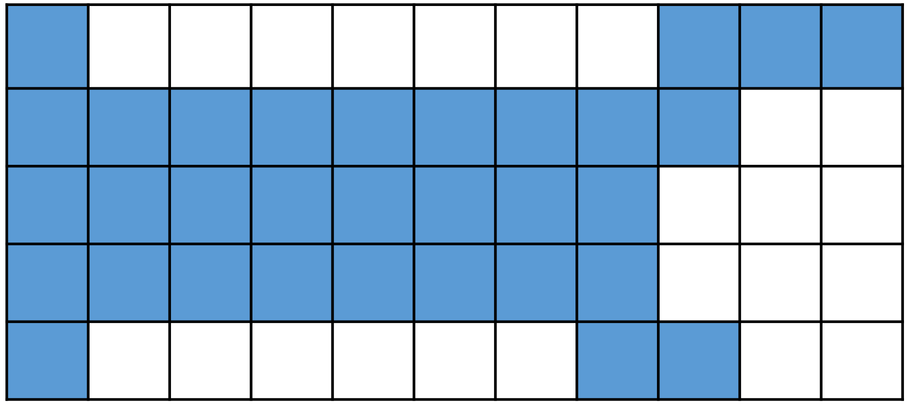
$$A_4 = A_3 \otimes B^4$$

Thinning

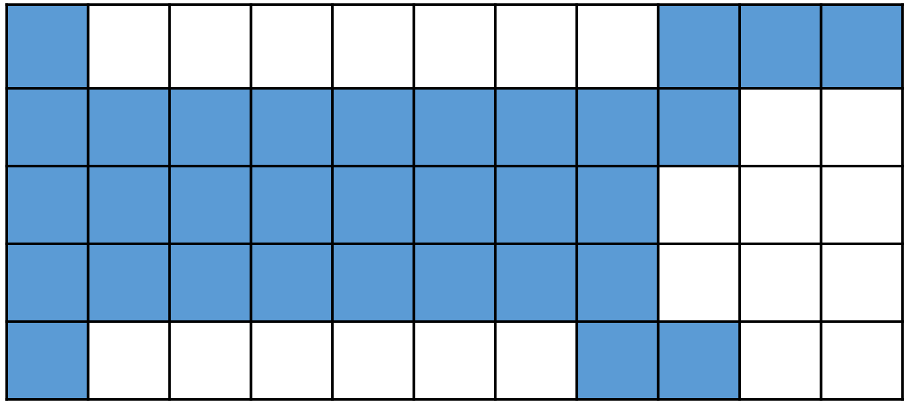


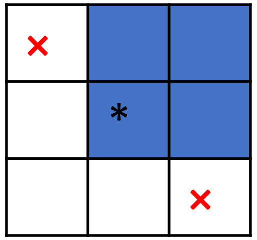


B^5

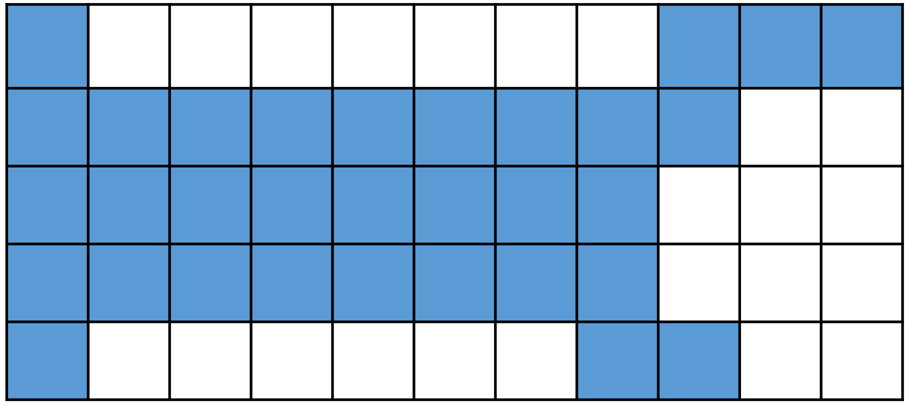


$$A_5 = A_4 \otimes B^5$$



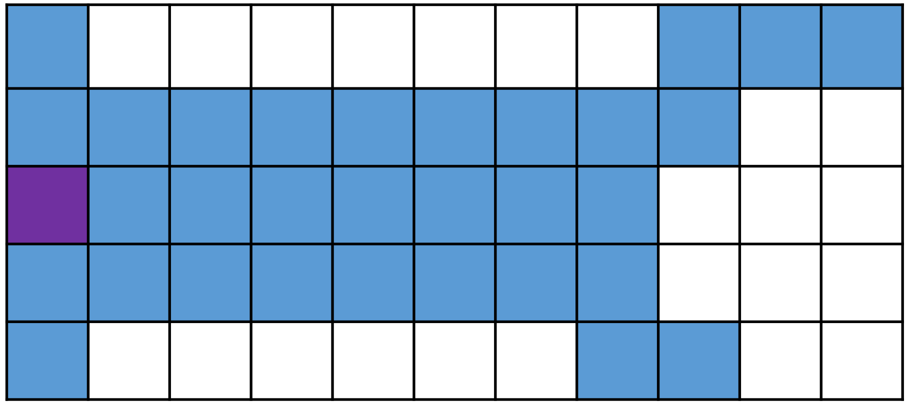


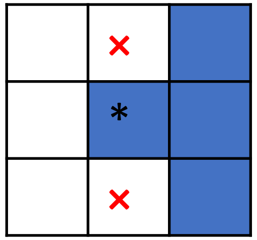
B^6



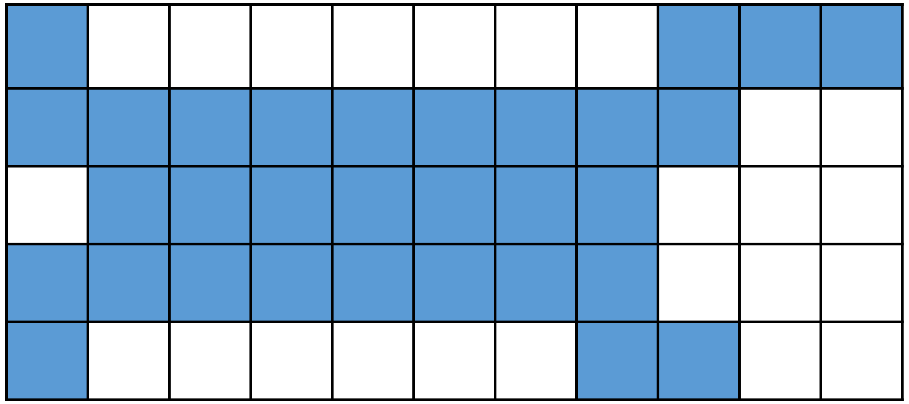
$$A_6 = A_5 \otimes B^6$$

Thinning

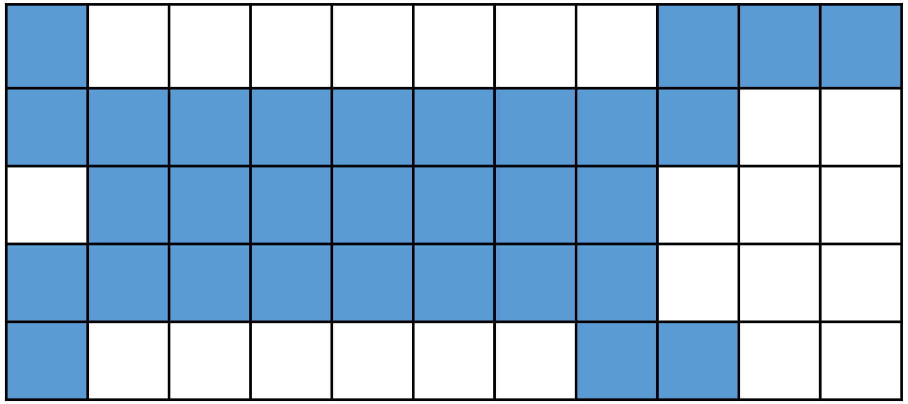


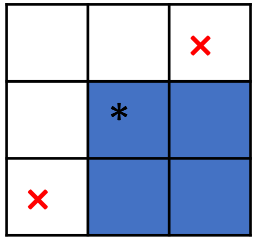


B^7

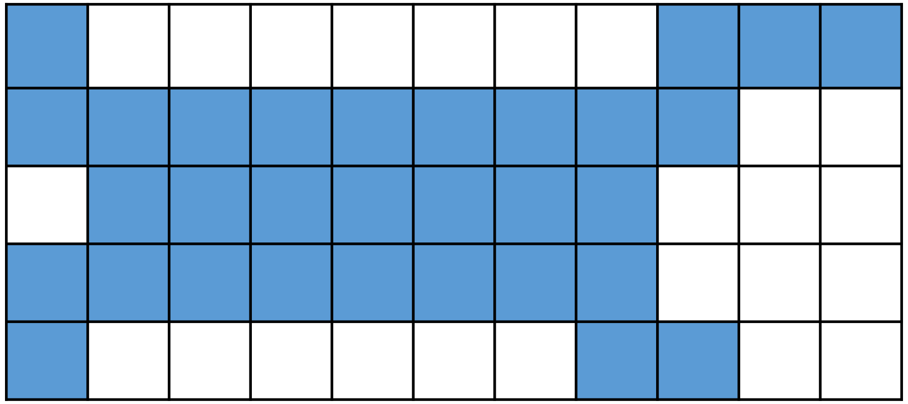


$A_8 = A_7 \otimes B^7$



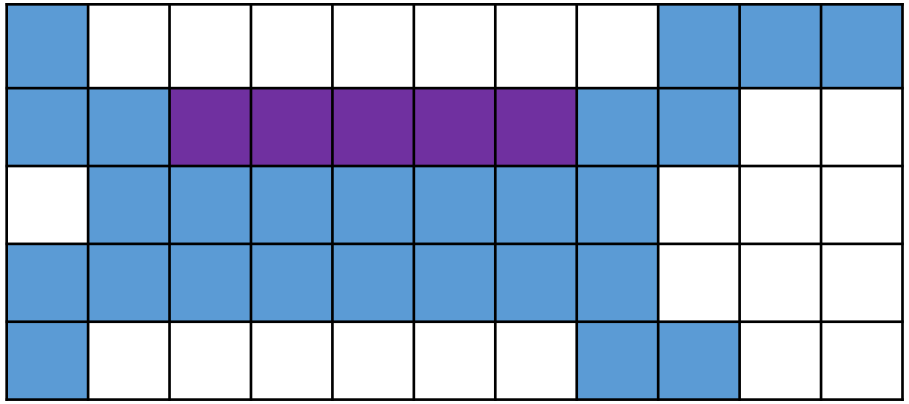


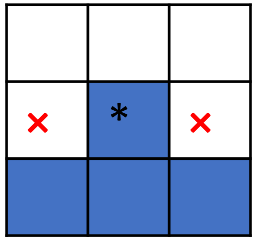
B^8



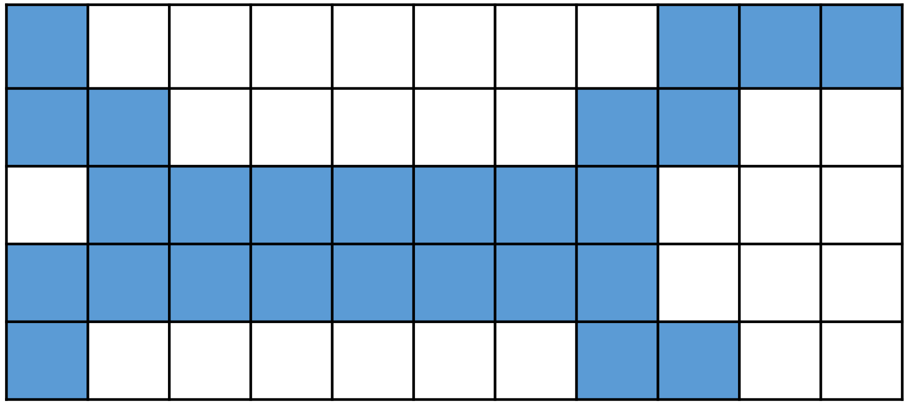
$A_9 = A_8 \otimes B^8$

Thinning

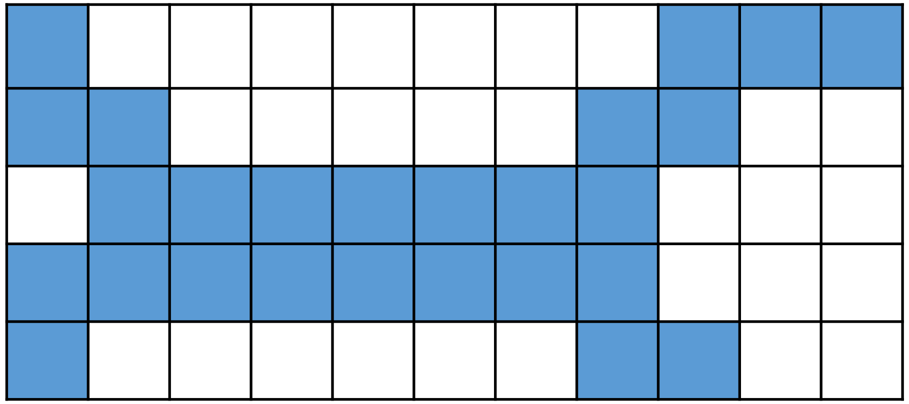


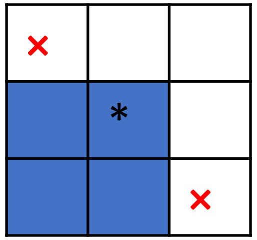


B^1

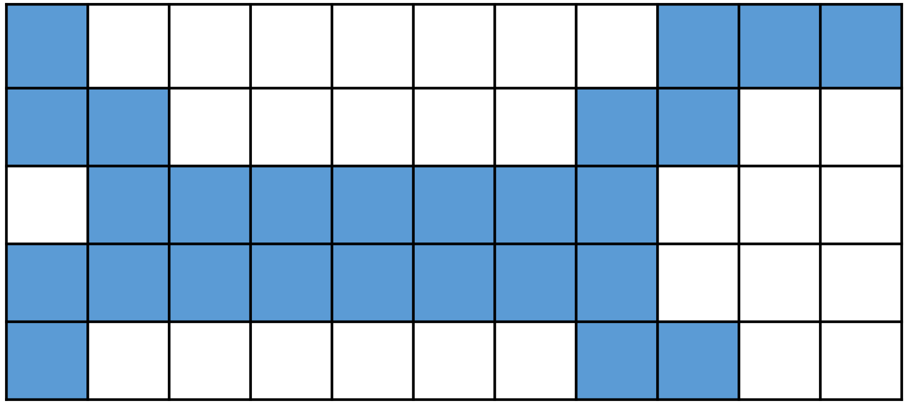


$A_{10} = A_9 \otimes B^1$



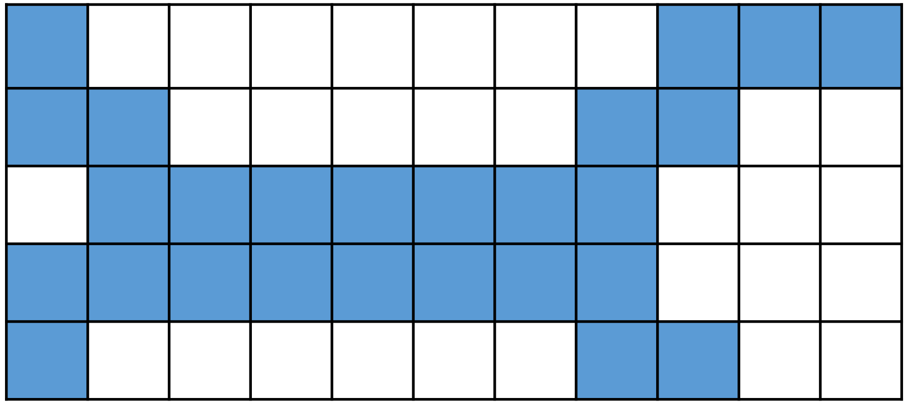


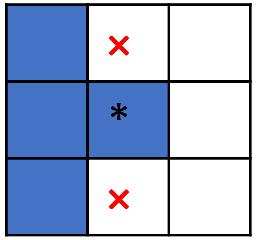
B^2



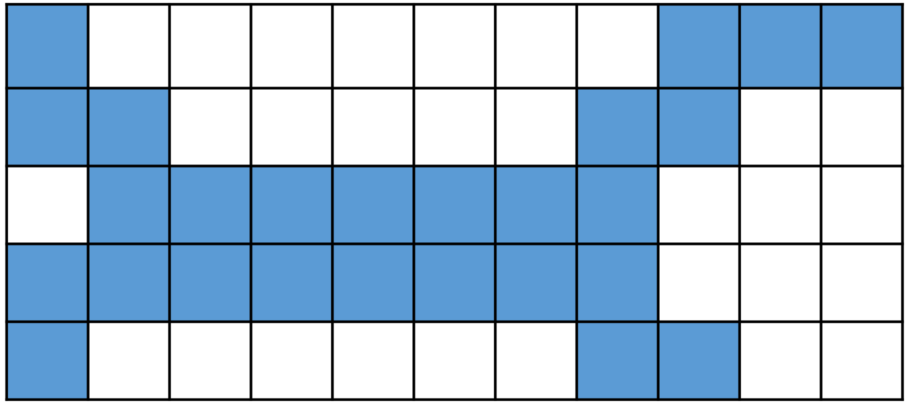
$A_{11} = A_{10} \otimes B^2$

Thinning

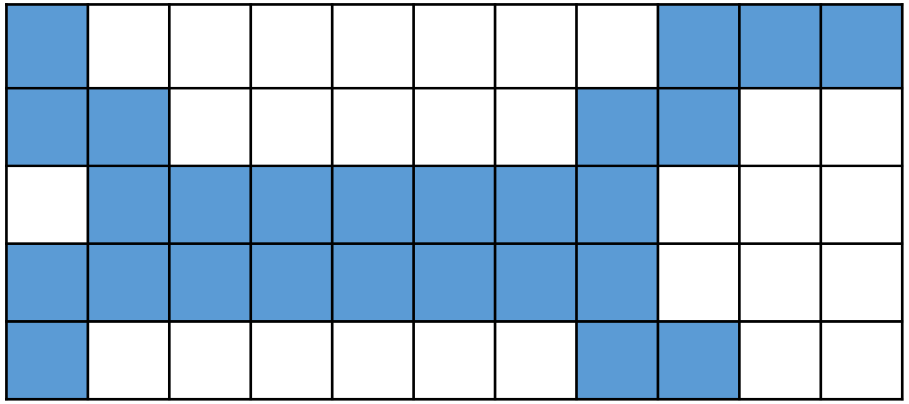


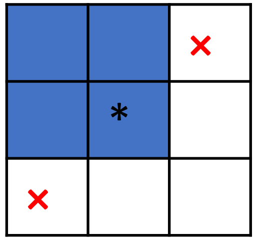


B^3

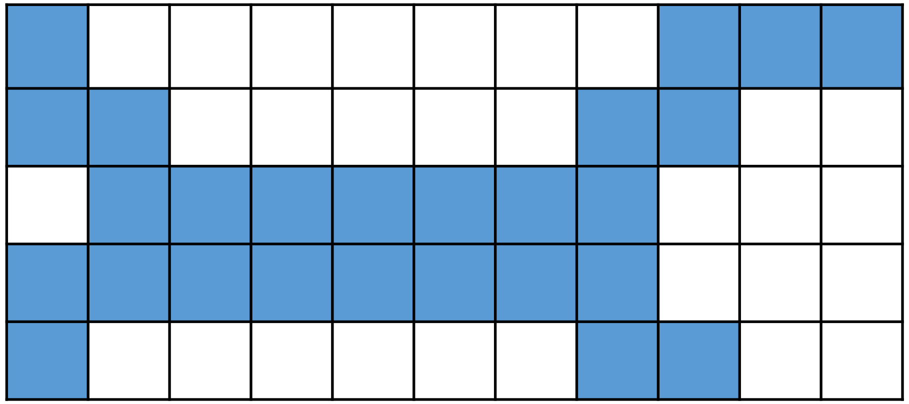


$A_{12} = A_{11} \otimes B^3$



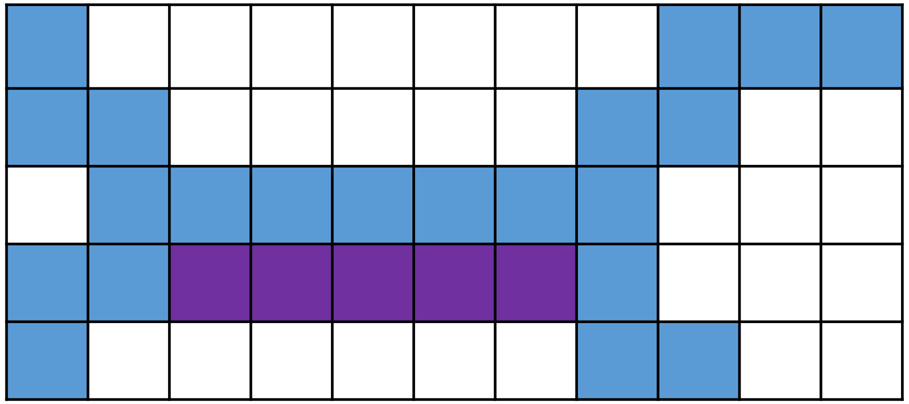


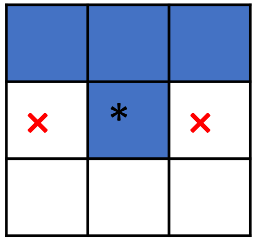
B^4



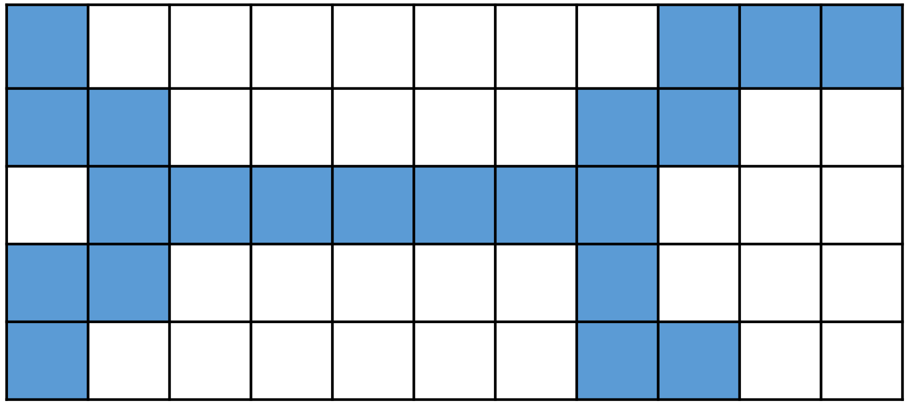
$A_{13} = A_{12} \otimes B^4$

Thinning



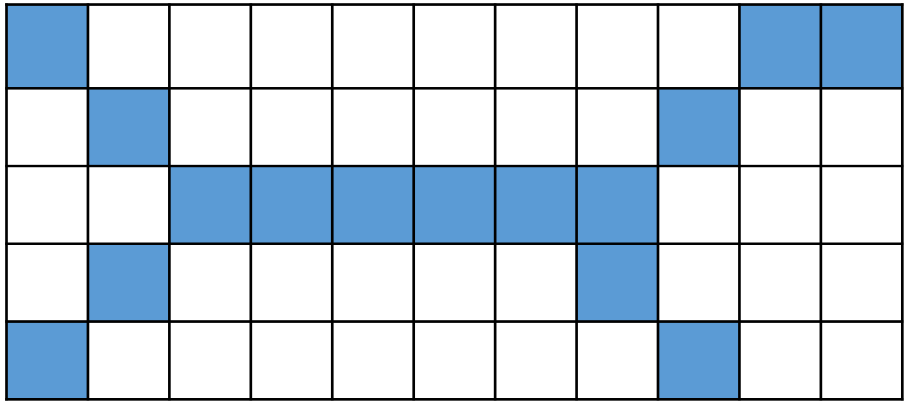


B^5



$A_{14} = A_{13} \otimes B^5$

No more change



converted to m-connectivity

Thickening

- Dual of Thinning $$A \odot B = A \cup (A \circledast B)$$

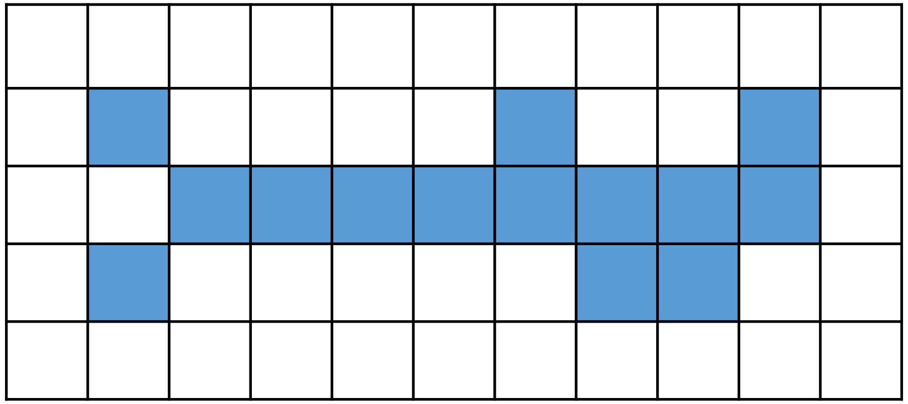
$$A \odot \{B\} = ((\dots((A \odot B^1) \odot B^2) \dots) \odot B^n)$$

- Operation : Calculate A^c

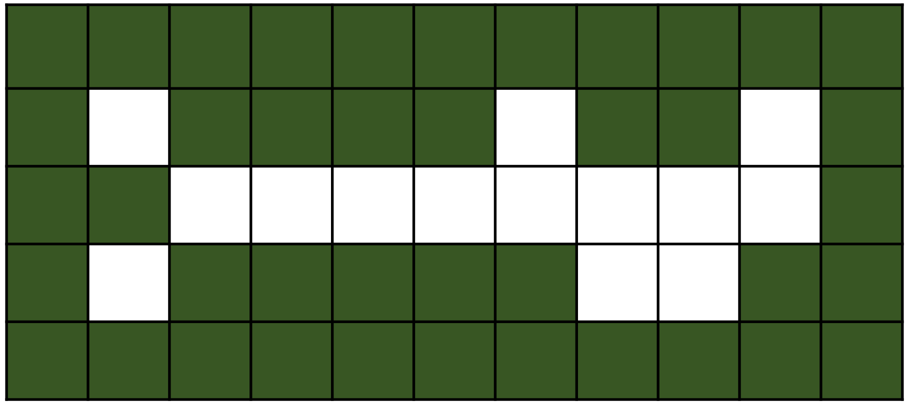
  Calculate thinning of A^c

  Then again calculate the complement of Thinning of A^c

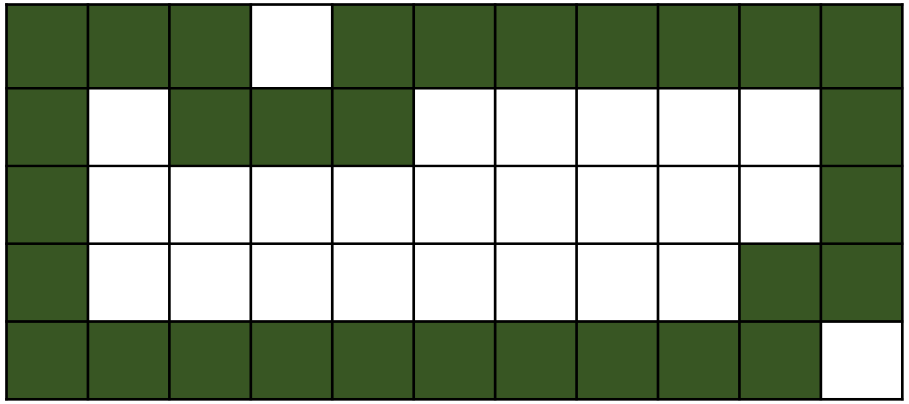
Thickening



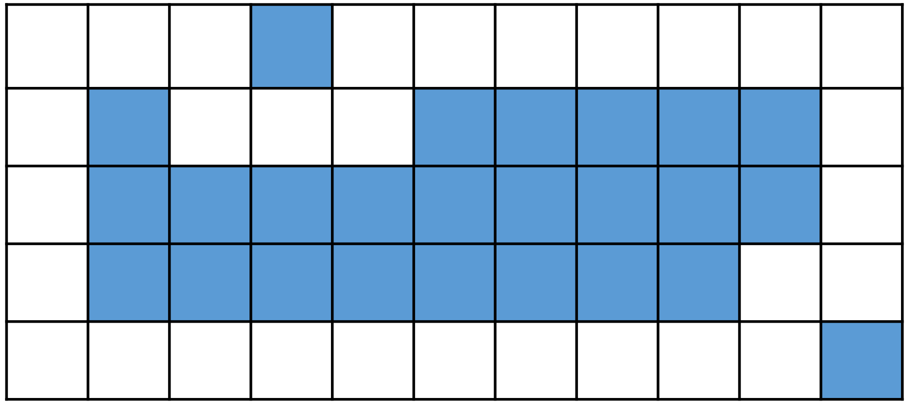
Set A



A^c

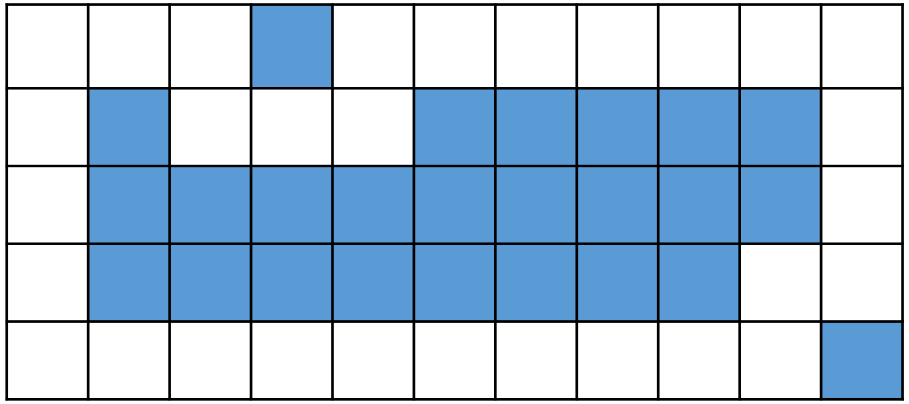


Thinning of A^c

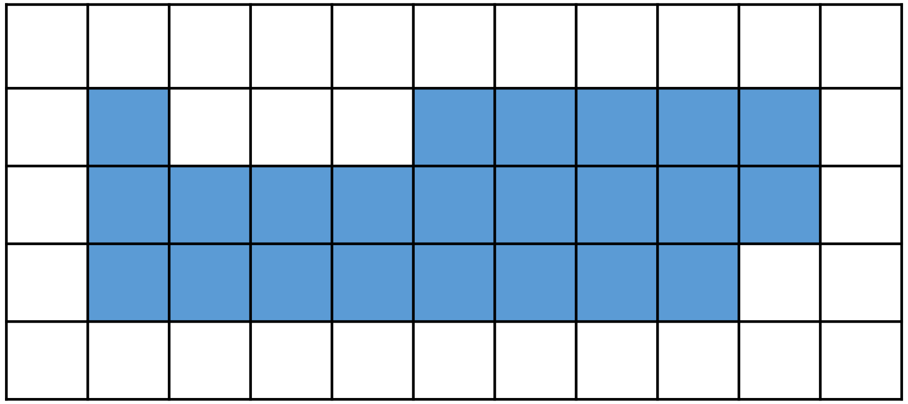


Complement of thinning A^c

Thickening



Thickening



final result with no disconnected points

Pruning

- Removing parasitic component
- Essential Complement to thinning and skeletonizing

$$X_1 = A \otimes \{B\}$$

$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$

$$X_3 = (X_2 \oplus H) \cap A$$

$$X_4 = X_1 \cup X_3$$

- **Structuring Elements**

$$B_1 = \begin{bmatrix} x & 0 & 0 \\ 1 & 1 & 0 \\ x & 0 & 0 \end{bmatrix} B_2 = \begin{bmatrix} x & 1 & x \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} B_3 = \begin{bmatrix} 0 & 0 & x \\ 0 & 1 & 1 \\ 0 & 0 & x \end{bmatrix} B_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ x & 1 & x \end{bmatrix}$$

$$B_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} B_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} B_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} B_8 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

‘x’ indicates a “don’t care” condition

Step 1: Thinning

- Apply this step a given (n) times to eliminate any branch with (n) or less pixels.

$$X_1 = A \otimes \{B\}$$

Step 2: Find End Points

Wherever the structuring elements are satisfied, the center of the 3x3 matrix is considered an endpoint.

$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$

Step 3: Dilate End Points

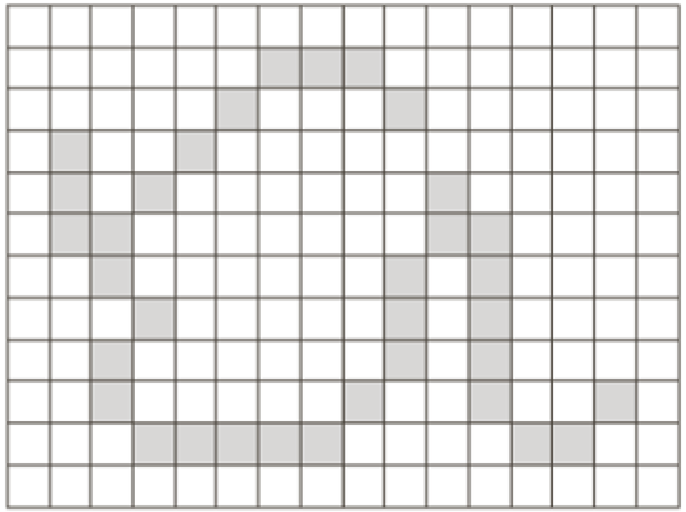
Perform dilation using a 3x3 matrix (H) consisting of all 1's and only insert 1's where the original image (A) also had a 1. Perform this for each endpoint in all direction (n) times.

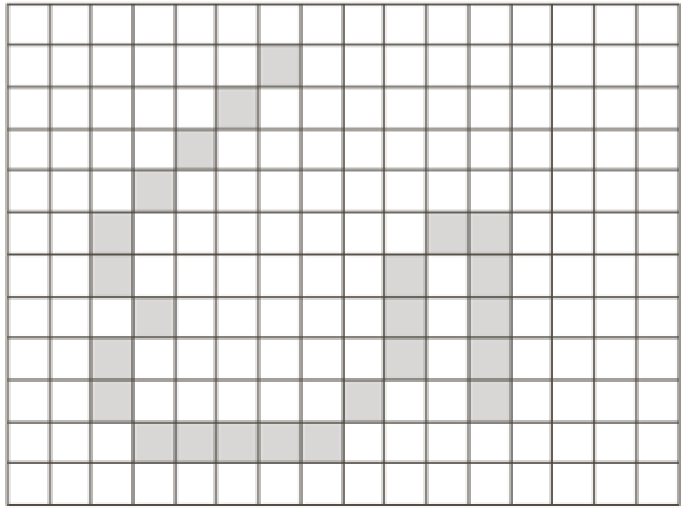
$$X_3 = (X_2 \oplus H) \cap A$$

Step 4: Union of X1 & X3

Take the result from step 1 and union it with step 3 to achieve the final results.

$$X_4 = X_1 \cup X_3$$

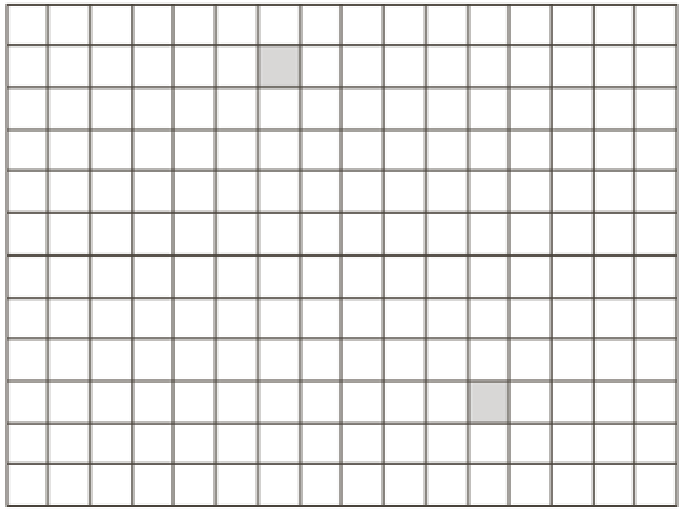




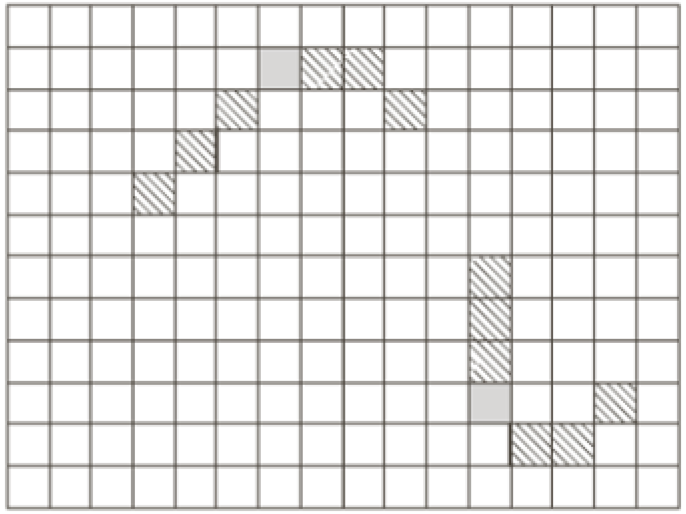
$$X_1 = A \otimes \{B\}$$

$$B_1 = \begin{bmatrix} x & 0 & 0 \\ 1 & 1 & 0 \\ x & 0 & 0 \end{bmatrix} B_2 = \begin{bmatrix} x & 1 & x \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} B_3 = \begin{bmatrix} 0 & 0 & x \\ 0 & 1 & 1 \\ 0 & 0 & x \end{bmatrix} B_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ x & 1 & x \end{bmatrix}$$

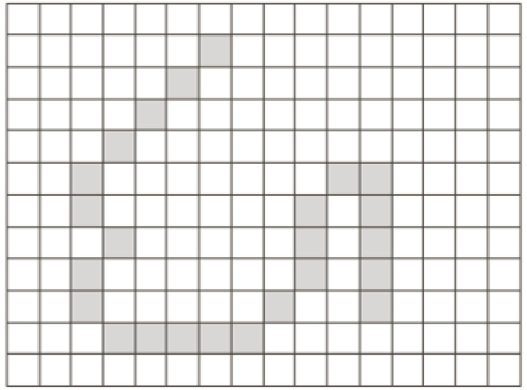
$$B_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} B_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} B_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} B_8 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$



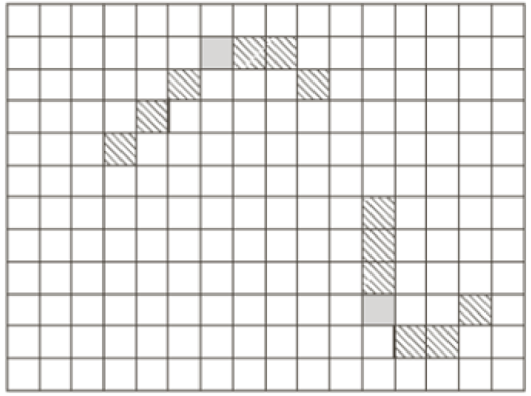
$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$



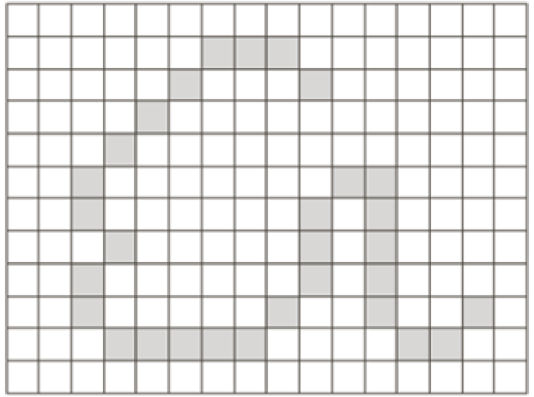
$$X_3 = (X_2 \oplus H) \cap A$$



$$X_1 = A \otimes \{B\}$$

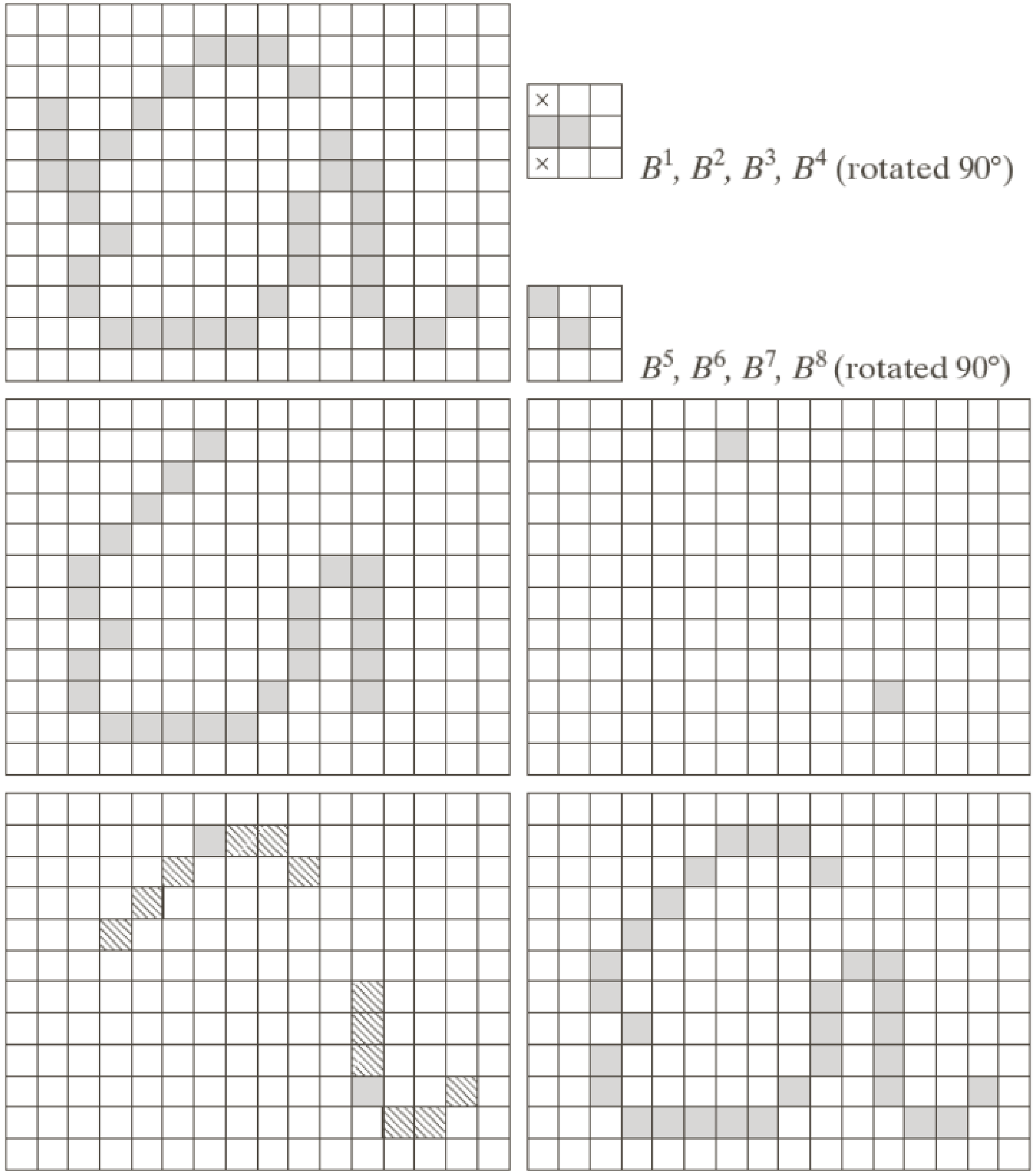


$$X_3 = (X_2 \oplus H) \cap A$$



$$X_4 = X_1 \cup X_3$$

Pruning



a b
c
d e
f g

FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

Morphological Reconstruction

- Morphological reconstruction that involves two images and a structuring element.
- One image, the ***marker***, contains the starting points for the transformation.
- The other image, the ***mask***, constrains the transformation. The structuring element is used to define connectivity

Geodesic dilation and erosion

- Let F denote the marker image and G the mask image.
- The *geodesic dilation* of size 1 of the marker image with respect to the mask

$$D_G^{(1)}(F) = (F \oplus B) \cap G$$

- The geodesic dilation of size n of F with respect G to is defined as

$$D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right]$$

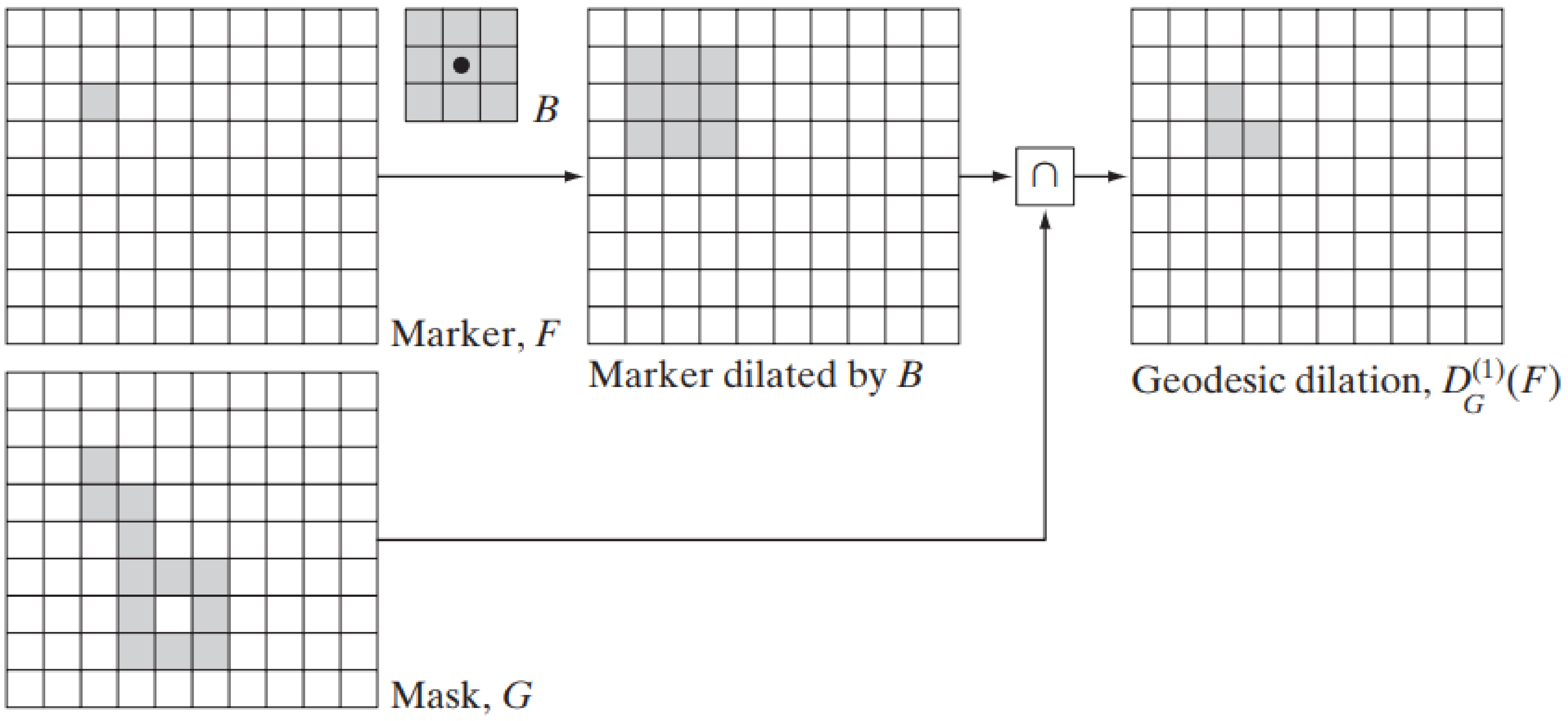


FIGURE 9.26 Illustration of geodesic dilation.

Geodesic erosion

$$E_G^{(1)}(F) = (F \ominus B) \cup G$$

$$E_G^{(n)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right]$$

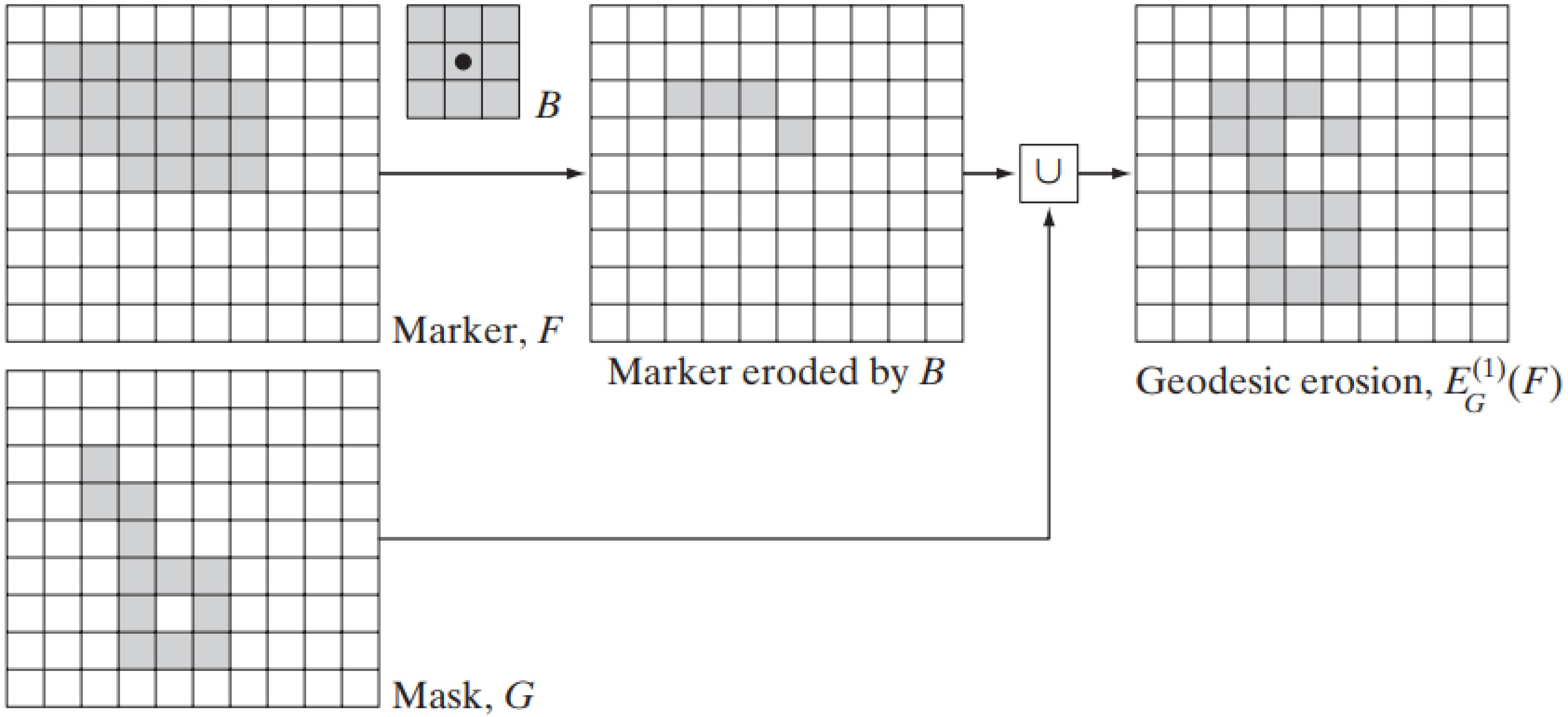


FIGURE 9.27 Illustration of geodesic erosion.

Morphological reconstruction by dilation and by erosion

- Based on the preceding concepts, *morphological reconstruction by dilation* of a mask image G from a marker image F

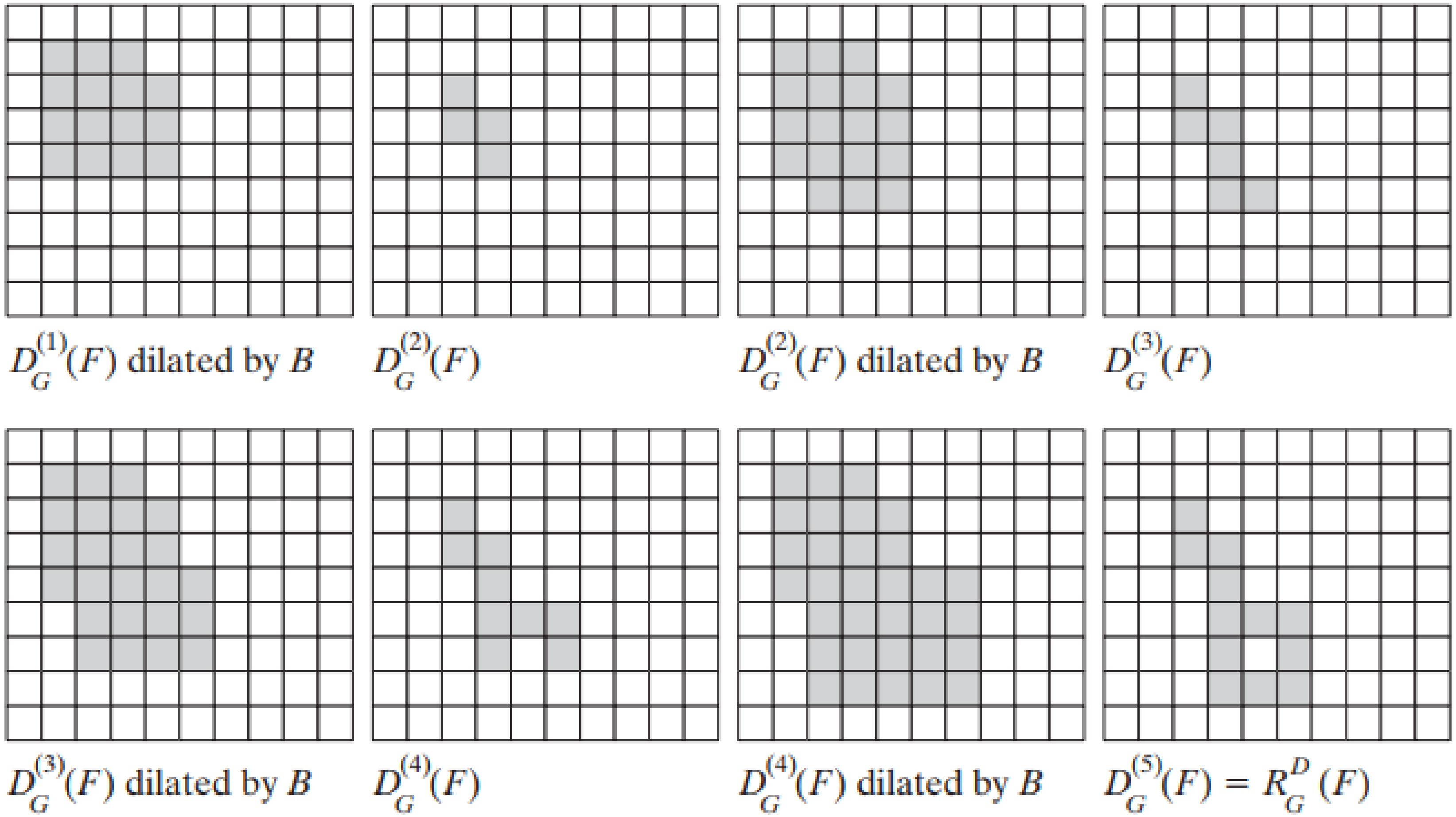
$$R_G^D(F) = D_G^{(k)}(F)$$

with k such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$.

- *Morphological reconstruction by erosion* of a mask image G from a marker image F.

$$R_G^E(F) = E_G^{(k)}(F)$$

with k such that $E_G^{(k)}(F) = E_G^{(k+1)}(F)$.



a b c d
e f g h

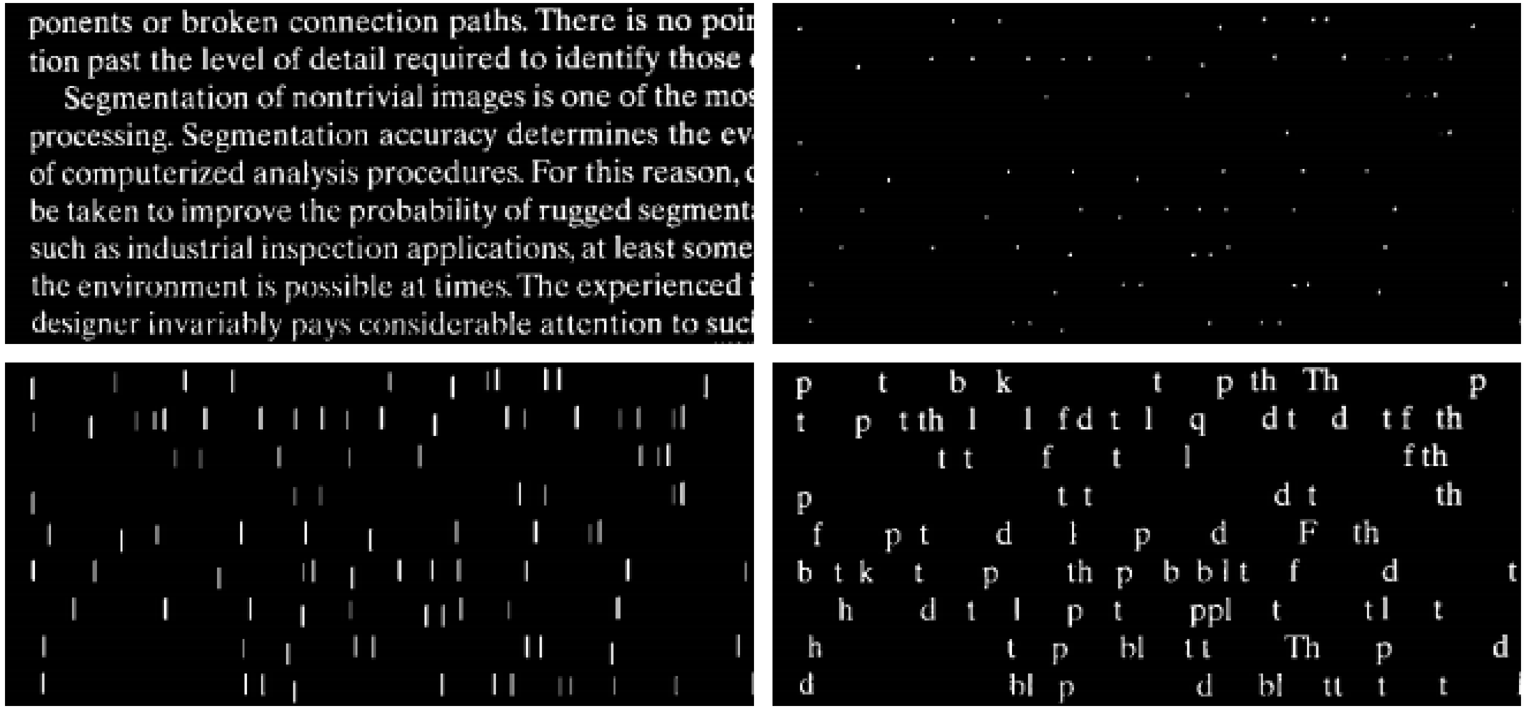
FIGURE 9.28 Illustration of morphological reconstruction by dilation. F, G, B and $D_G^{(1)}(F)$ are from Fig. 9.26.

Opening by reconstruction:

- The opening by reconstruction of size n of an image F is defined as the reconstruction by dilation of F from the erosion of size n of F that is,

$$O_R^{(n)}(F) = R_F^D\left[(F \ominus nB)\right]$$

where $(F \ominus nB)$ indicates n erosions of F by B,



a b
c d

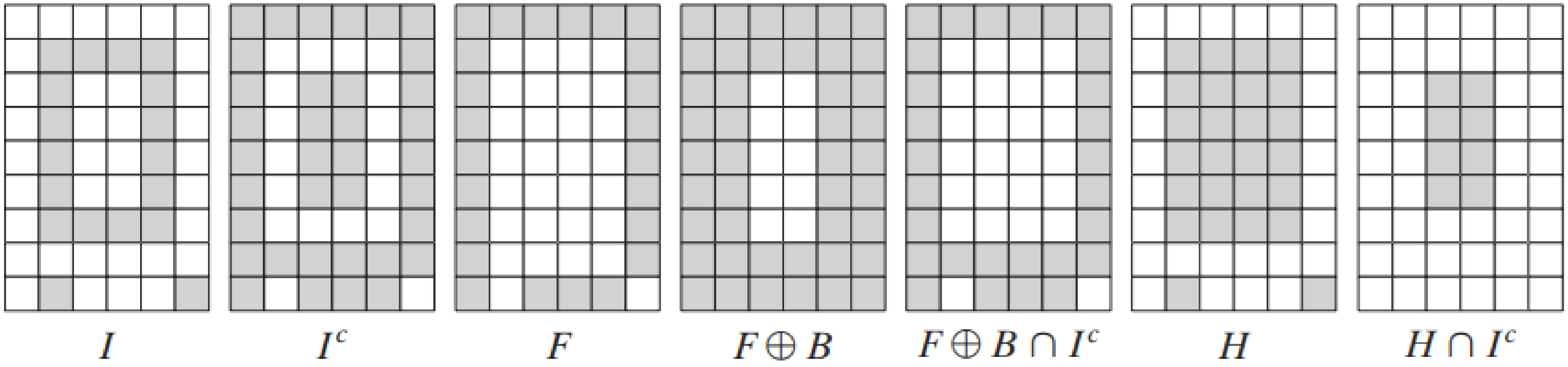
FIGURE 9.29 (a) Text image of size 918 × 2018 pixels. The approximate average height of the tall characters is 50 pixels. (b) Erosion of (a) with a structuring element of size 51 × 1 pixels. (c) Opening of (a) with the same structuring element, shown for reference. (d) Result of opening by reconstruction.

Filling holes:

$$F(x, y) = \begin{cases} 1 - I(x, y) & \text{if } (x, y) \text{ is on the border of } I \\ 0 & \text{otherwise} \end{cases}$$

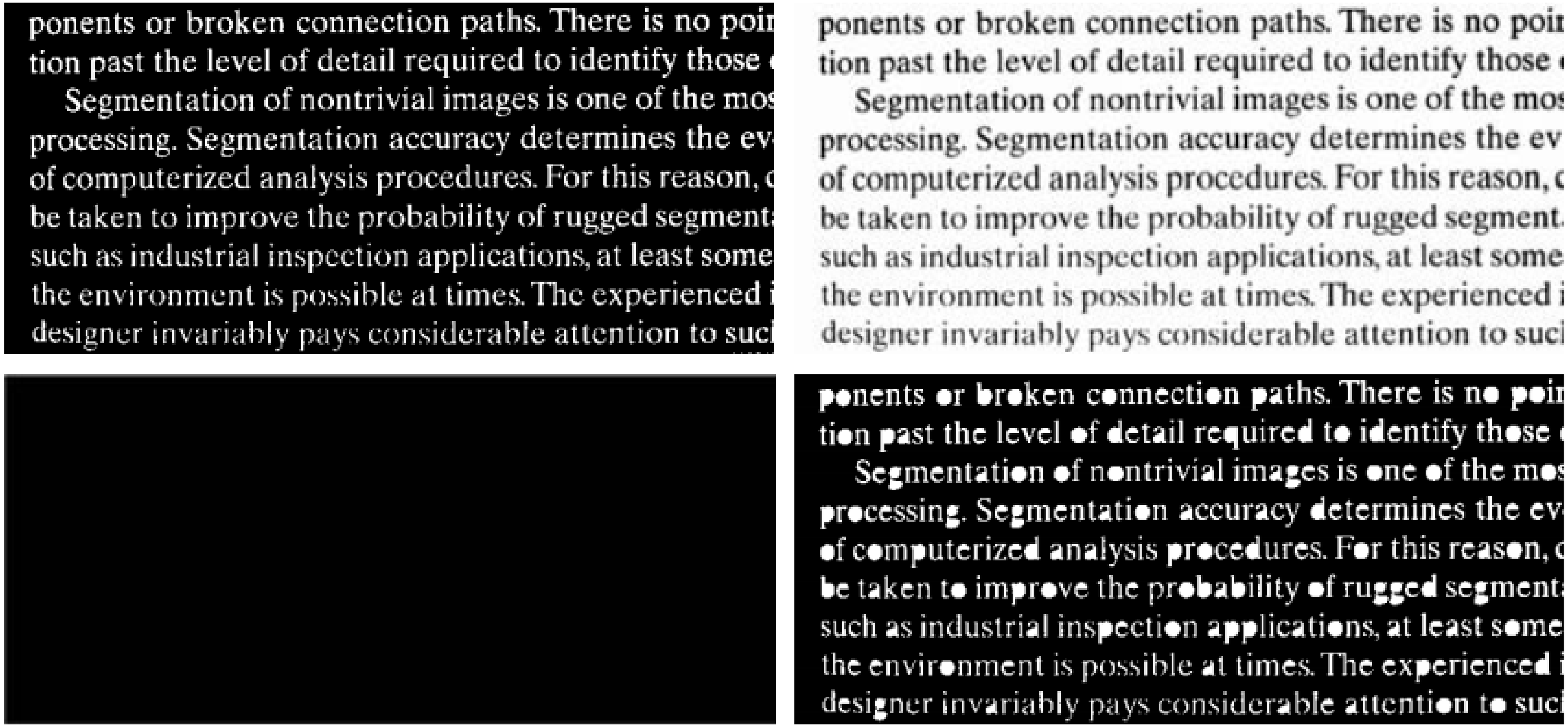
Then

$$H = \left[R_{I^c}^{D}(F) \right]^c$$



a b c d e f g

FIGURE 9.30 Illustration of hole filling on a simple image.



a b
c d

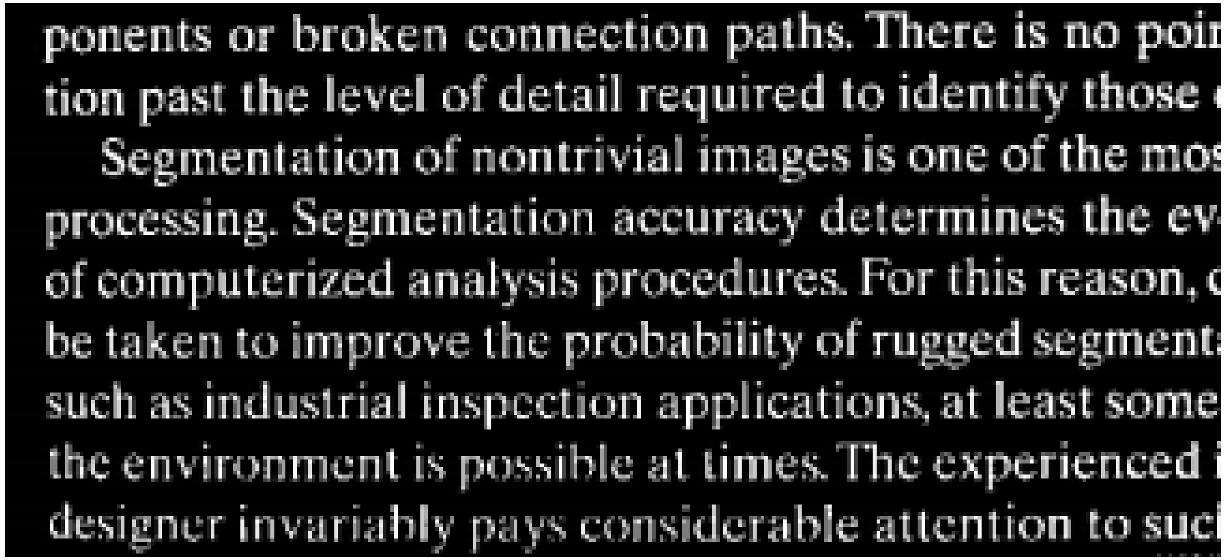
FIGURE 9.31
(a) Text image of size 918 × 2018 pixels. (b) Complement of (a) for use as a mask image. (c) Marker image. (d) Result of hole-filling using Eq. (9.5-29).

Border clearing:

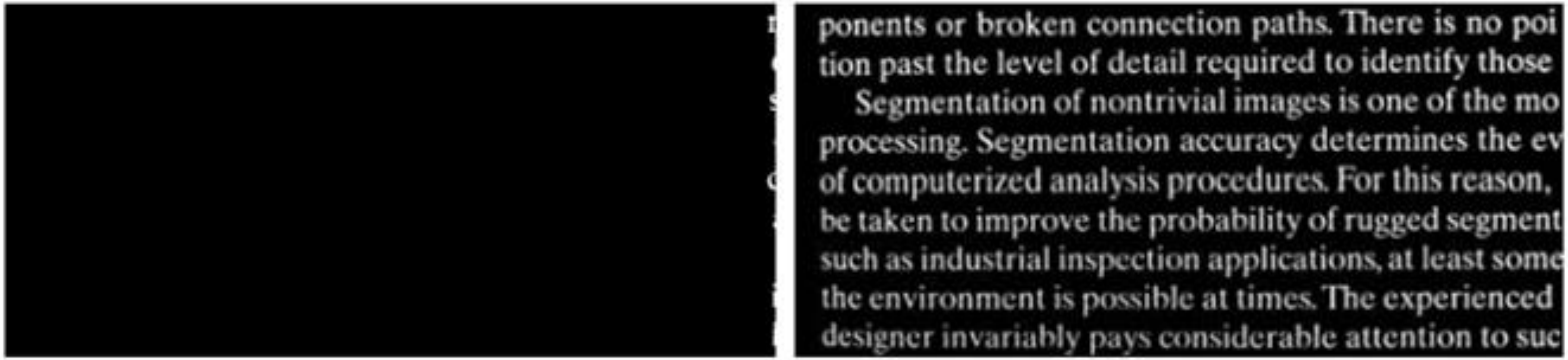
- In this application, we use the original image as the mask and the following marker image:

$$F(x, y) = \begin{cases} I(x, y) & \text{if } (x, y) \text{ is on the border of } I \\ 0 & \text{otherwise} \end{cases}$$

$$X = I - R_I^D(F)$$



ponents or broken connection paths. There is no poi
tion past the level of detail required to identify those
Segmentation of nontrivial images is one of the mos
processing. Segmentation accuracy determines the ev
of computerized analysis procedures. For this reason,
be taken to improve the probability of rugged segment
such as industrial inspection applications, at least some
the environment is possible at times. The experienced
designer invariably pays considerable attention to suc



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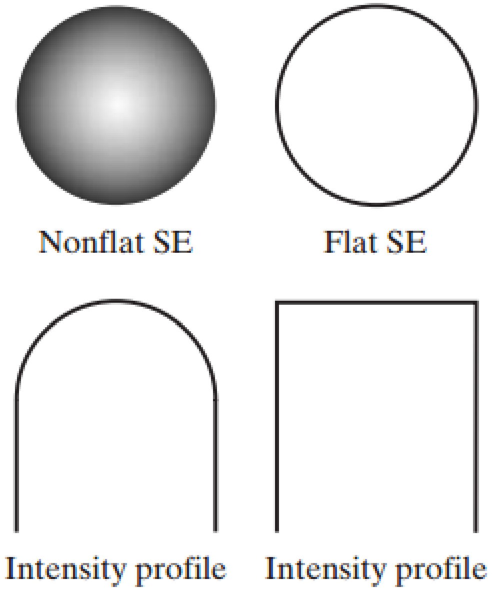
such as industrial inspection applications, at least some
the environment is possible at times. The experienced
designer invariably pays considerable attention to suc

Structuring elements

- Structuring elements in gray-scale morphology belong to one of two categories: nonflat and flat.



a b
c d

FIGURE 9.34 Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their center. All examples in this section are based on flat SEs.

Erosion for Gray scale Image

- The **erosion** of **f** by a flat structuring element **b** at any location (x,y) is defined as the **minimum** value of the image in the region coincides with b when the origin of b is at (x,y).

$$[f \ominus b](x,y) = \min_{(s,\, t) \in b} \{ f(x+s,\ y+t) \}$$

- The **erosion** of **f** by a non flat structuring element

$$[f \ominus b_N](x,y) = \min_{(s,\, t) \in b_N} \{ f(x+s,\ y+t) - b_N(s,\, t) \}$$

Erosion– Gray-Scale

- General effect of performing an erosion in grayscale images:

  1. If all elements of the structuring element are positive, the output image tends to be **darker** than the input image.

  1. The effect of **bright details** in the input image that are smaller in area than the structuring element is **reduced**, with the degree of reduction being determined by the grayscale values surrounding by the bright detail and by shape and amplitude values of the structuring element itself.

Dilation – Gray-Scale

- The **Dilation** of **f** by a flat structuring element **b** at any location (x,y) is defined as the **maximum** value of the image in the region outlined by when the origin of is at (x,y).

$$[f \oplus b](x,y) = \max_{(s,t) \in \hat{b}} \{f(x-s, y-t)\}$$

- The **Dilation** of **f** by a non flat structuring element

$$[f \oplus b_N](x, y) = \max_{(s, t) \in \hat{b}_N} \{f(x-s, y-t) + \hat{b}_N(s, t)\}$$

Dilation – Gray-Scale

- The general effects of performing dilation on a gray scale image is twofold:

1. If all the values of the structuring elements are positive than the output image tends to be **brighter** than the input.

1. **Dark details** either are **reduced** or **eliminated**, depending on how their values and shape relate to the structuring element used for dilation

Erosion and Dilation for Gray scale image

- Erosion and dilation are duals.

$$(f \ominus b)^c = f^c \oplus \hat{b}$$

$$(f \oplus b)^c = f^c \ominus \hat{b}$$

Opening and Closing for Gray scale Image

- Opening – $f \circ b = (f \ominus b) \oplus b.$
- Closing – $f \bullet b = (f \oplus b) \ominus b.$

The opening and closing for grayscale images are duals with respect to complementation and SE reflection:

$$(f \bullet b)^c = f^c \circ \hat{b}$$

$$(f \circ b)^c = f^c \bullet \hat{b}$$

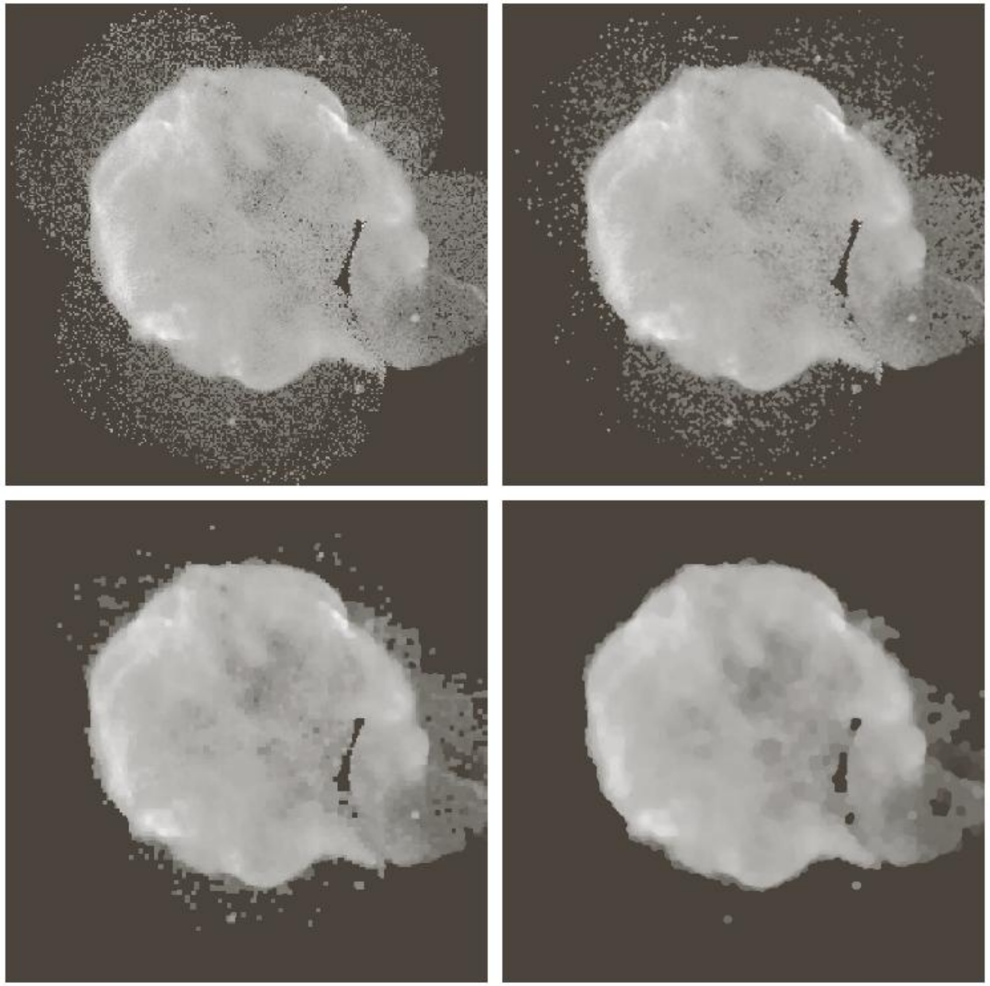
Some basic Gray-Scale Morphological Algorithms

- Morphological Smoothing
- Morphological gradient
- Top-hat and bottom-hat transformations
- Granulometry
- Textural segmentation

Morphological Smoothing

- A procedure used named **alternating sequential filtering**, in which the opening–closing sequence starts with the original image
- Subsequent steps perform the opening and closing on the results of the previous step.
- Useful in automated image analysis, in which results at each step are compared against a specified metric.

Morphological Smoothing



a b
c d

FIGURE 9.38 (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

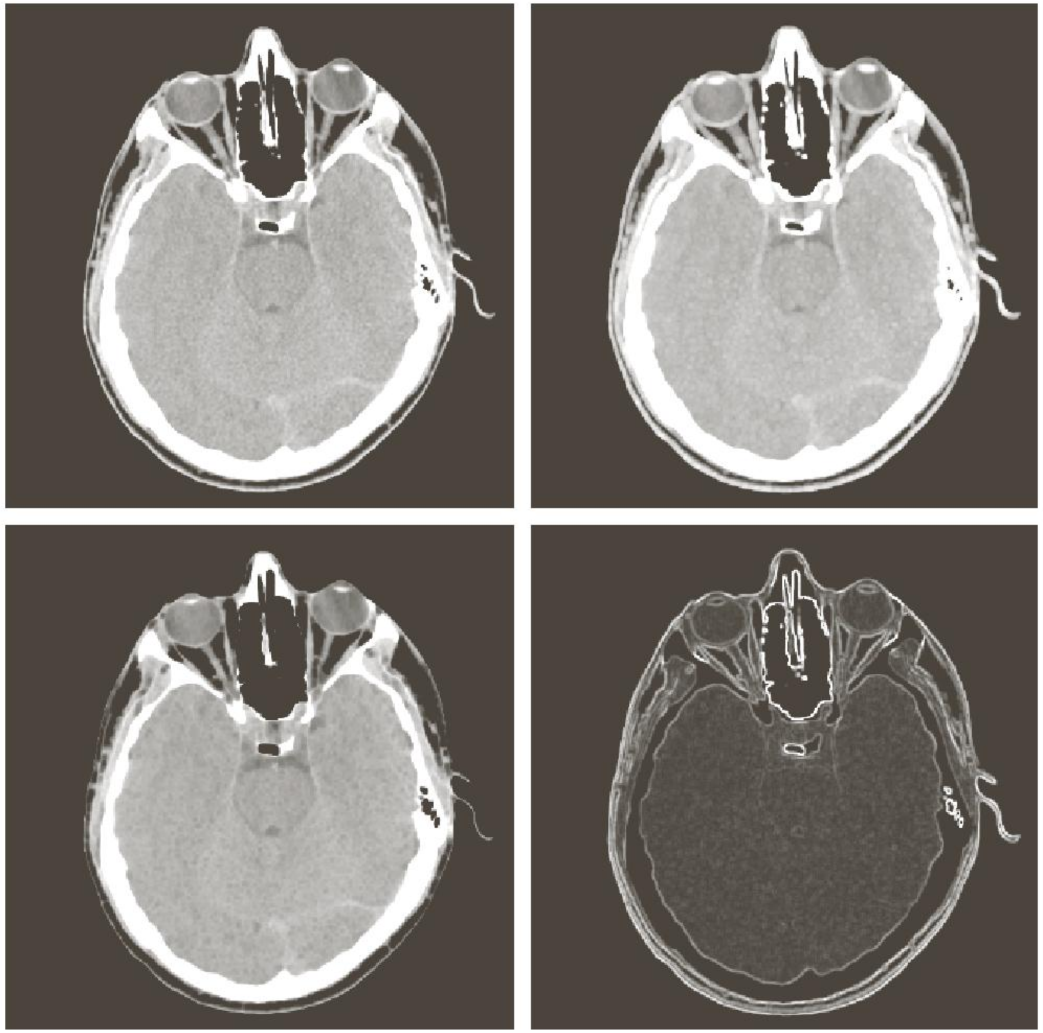
Morphological gradient

- Dilation and Erosion can be used in combination with image subtraction to obtain the morphological gradient of an image.

$$g = (f \oplus b) - (f \theta b)$$

- Emphasizes the boundaries between regions.
- Homogenous areas are not affected.
- The net result is an image in which the edges are enhanced and the contribution of the homogeneous areas is suppressed, thus producing a “derivative-like” (gradient) effect.

Morphological gradient



a b
c d

FIGURE 9.39
(a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

Top-hat and bottom-hat transformations

- Combining image subtraction with openings and closings

$$T_{hat}(f) = f - (f \circ b)$$
$$B_{hat}(f) = (f \bullet b) - f$$

- The top-hat transformation is used for light objects on a dark background, the bottom-hat transformation is used for the opposite situation.
- Removing objects from image by using structuring element
- Correcting the effect of non-uniform illumination.
- Great role in Segmentation

Top-hat and bottom-hat transformations

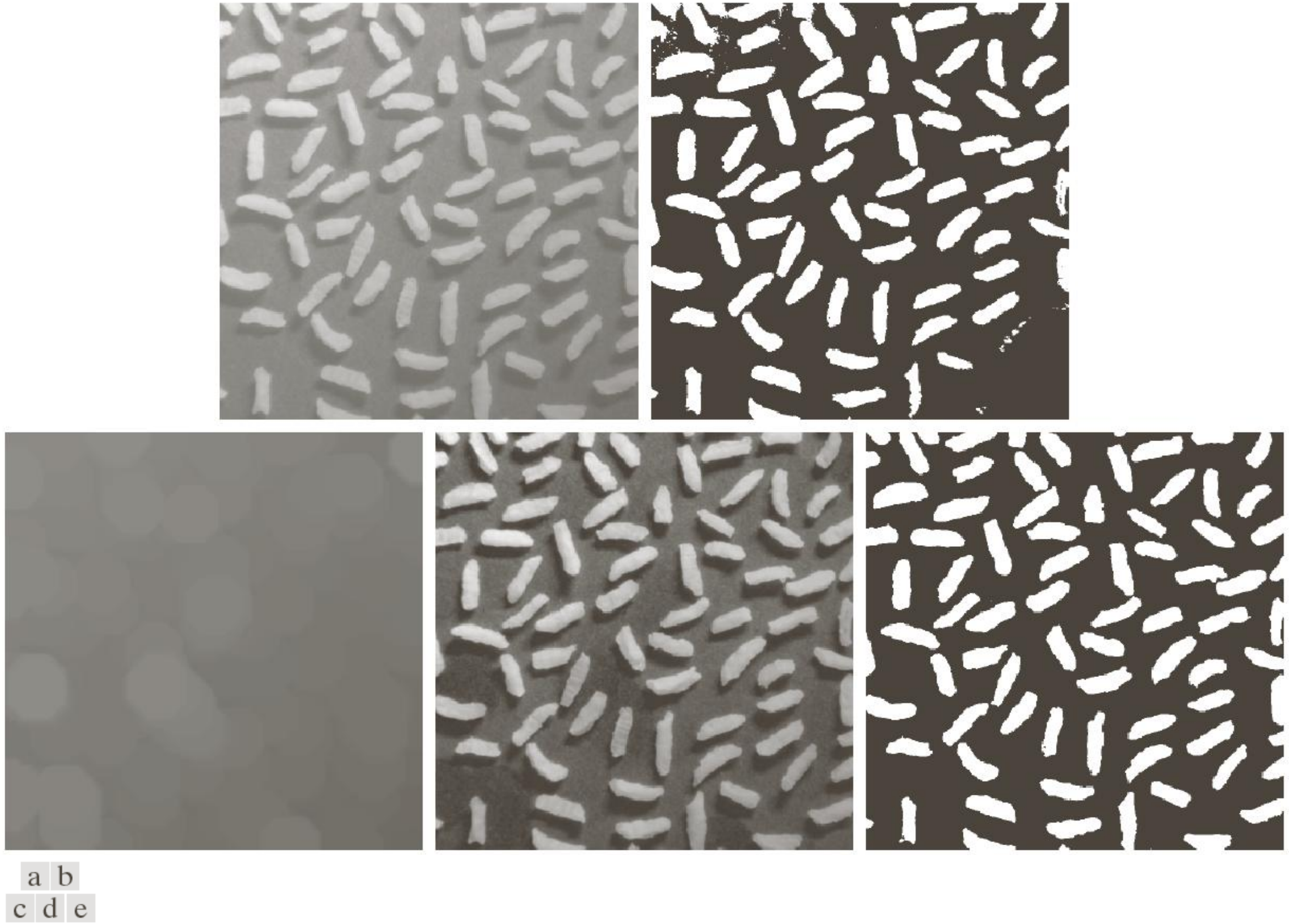


FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size 600×600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

Granulometry

- Determining the size distribution of particles in an image
- Consists of applying openings with SEs of increasing size.

Granulometry

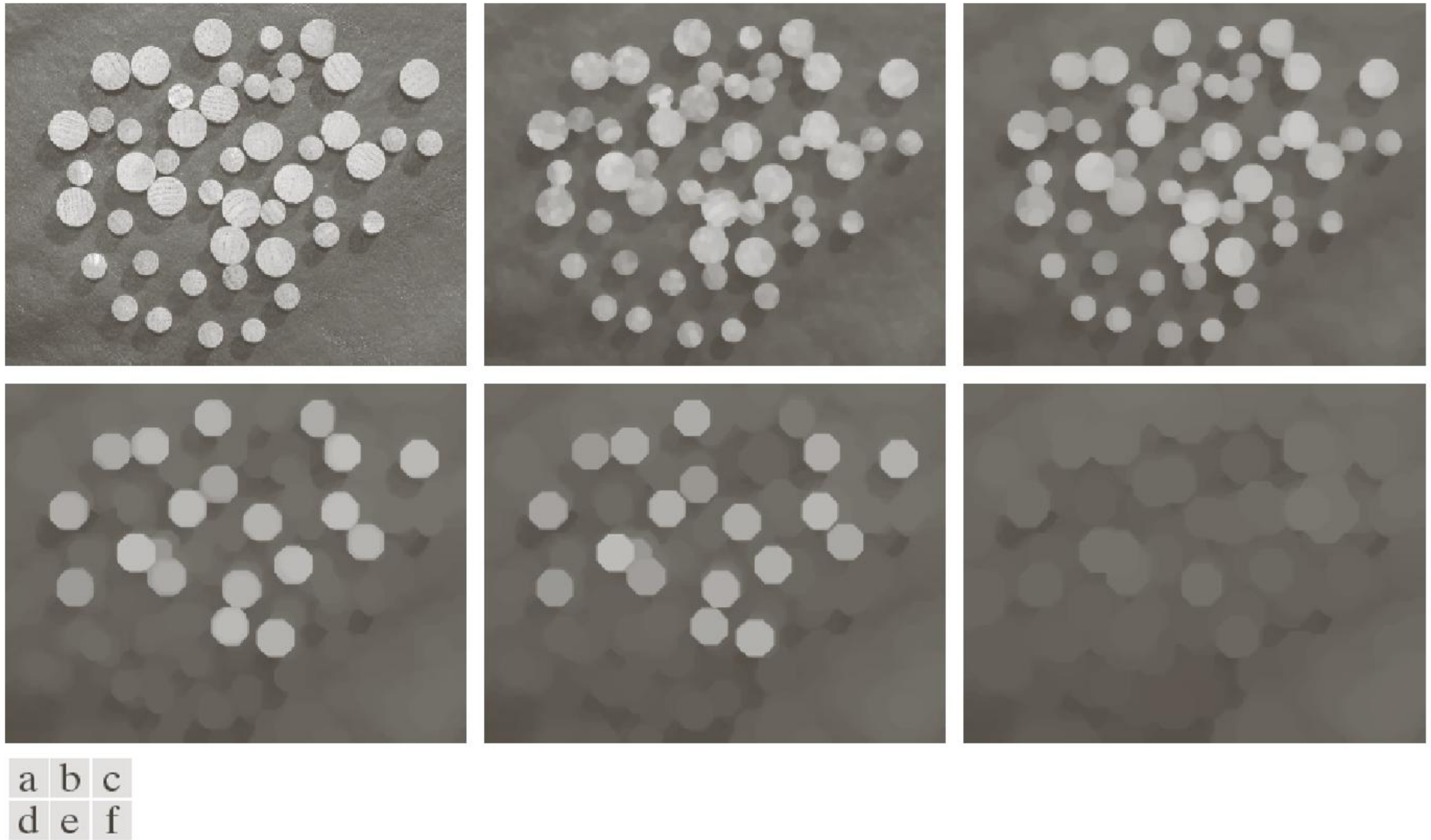
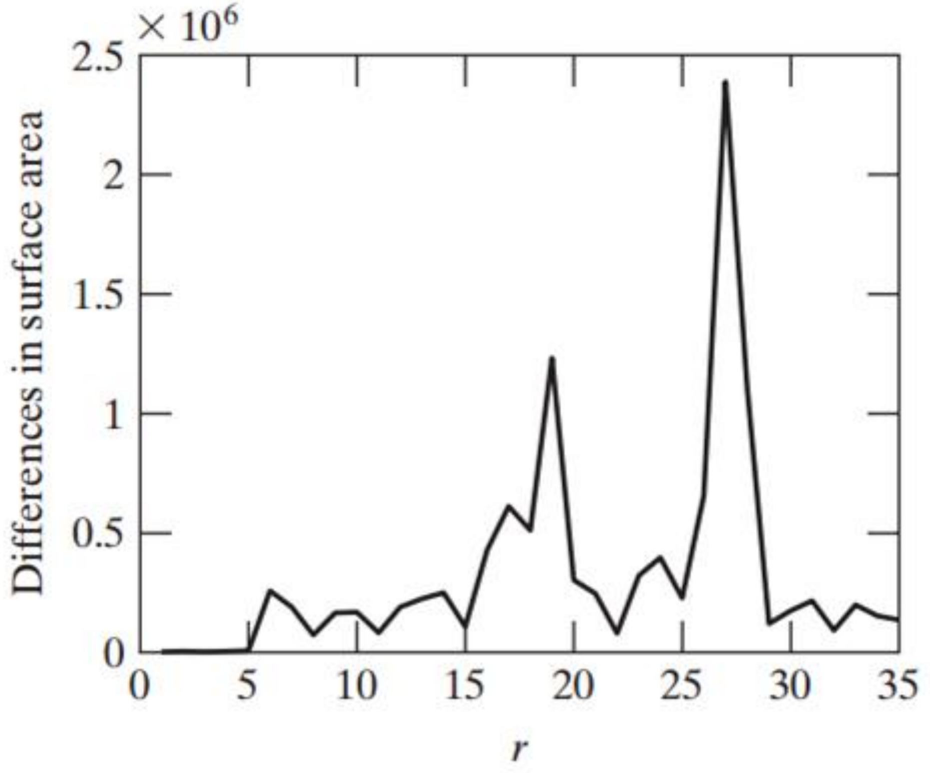


FIGURE 9.41 (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

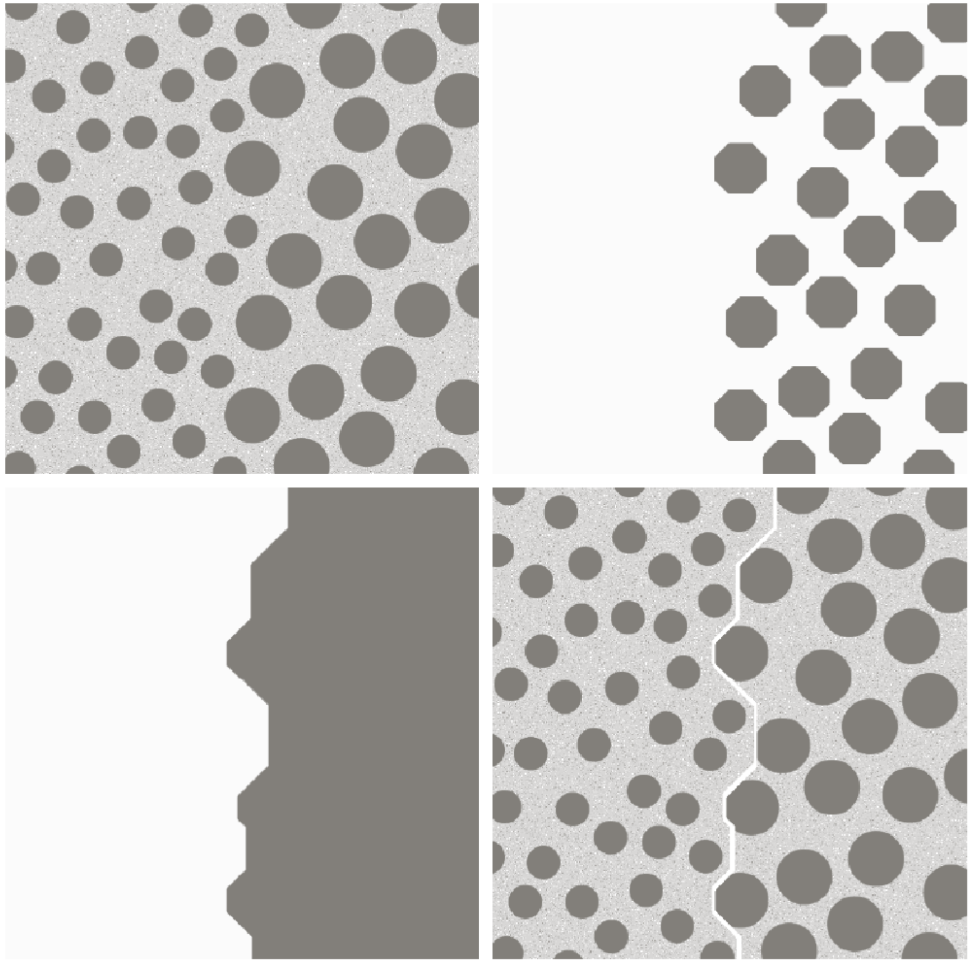


- Opening operation done.
- Sum of the pixel values is computed. This sum, called the surface area.
- This procedure yields a 1-D array each element of which is the sum of the pixels in the opening for the size SE corresponding to that location in the array.
- The difference between adjacent elements of the 1-D array are plotted, the peaks in the plot are an indication of the predominant size distributions of the particles in the image

Textural segmentation

- The objectives is to find a boundary between two regions based on their textural content.
- Using closings and openings

Textural segmentation



a b
c d

FIGURE 9.45 Textural segmentation. (a) A 600 × 600 image consisting of two types of blobs. (b) Image with small blobs removed by closing (a). (c) Image with light patches between large blobs removed by opening (b). (d) Original image with boundary between the two regions in (c) superimposed. The boundary was obtained using a morphological gradient.

GRAYSCALE MORPHOLOGICAL RECONSTRUCTION

The **geodesic dilation** of size 1 of f with respect to g is defined as

$$D_g^{(1)}(f) = (f \oplus b) \wedge g$$

The geodesic dilation of size n of f with respect to g is defined as

$$D_g^{(n)}(f) = D_g^{(1)}\left(D_g^{(n-1)}(f)\right)$$

GRAYSCALE MORPHOLOGICAL RECONSTRUCTION

The geodesic erosion of size 1 of f with respect to g is defined as

$$E_g^{(1)}(f) = (f \ominus b) \vee g$$

The geodesic erosion of size n is defined as

$$E_g^{(n)}(f) = E_g^{(1)}\left(E_g^{(n-1)}(f)\right)$$

GRAYSCALE MORPHOLOGICAL RECONSTRUCTION

The *morphological reconstruction by dilation* of a *grayscale mask image*, g, by a *grayscale marker image*, f, denoted by $R_g^{(D)}(f)$, is defined as the geodesic dilation of f with respect to g, iterated until stability is reached; that is,

$$R_g^D(f) = D_g^{(k)}(f)$$

with k such that $D_g^{(k)}(f) = D_g^{(k+1)}(f)$. The *morphological reconstruction by erosion* of g by f, denoted by $R_g^E(f)$, is similarly defined as

$$R_g^E(f) = E_g^{(k)}(f)$$

with k such that $E_g^{(k)}(f) = E_g^{(k+1)}(f)$.

End